IBSL Functions Past Paper Questions SOLUTIONS

1. (a) **METHOD 1**

$f(3) = \sqrt{7}$ (A1)

$(g \circ f)(3) = 7$ A1 N2

METHOD 2

$(g \circ f)(x) = \sqrt{x+4}^2 \quad (= x + 4)$ (A1)

$(g \circ f)(3) = 7$ A1 N2

(b) For interchanging x and y (seen anywhere) (M1)
Evidence of correct manipulation A1

eg $x = \sqrt{y+4}, x^2 = y+4$

$f^{-1}(x) = x^2 - 4$ A1 N2

(c) $x \geq 0$ A1 N1

[6]

2. (a) $f^{-1}(2) \Rightarrow 3x + 5 = 2$ (M1)
$x = -1$ (A1) (C2)

(b) $g(f(-4) = g(-12 + 5)$
$= g(-7)$ (A1)
$= 2(1 + 7)$
$= 16$ (A1) (C2)

[4]

3. (a) **METHOD 1**

$(f \circ g)(4) = f(g(4)) = f(1)$ (M1)

$= 2$ (A1) (C2)

METHOD 2

$(f \circ g)(x) = \dfrac{2}{x-3}$ (M1)

$(f \circ g)(4) = 2$ (A1) (C2)

(b) Let $y = \dfrac{1}{x-3}$

Correct simplification $y(x-3) = 1$ $\left(x - 3 = \dfrac{1}{y}\right)$ (A1)

$$x = \frac{1}{y} + 3 \quad \left(= \frac{1+3y}{y}\right)$$ (A1)

Interchanging x and y (may happen earlier) (M1)

$y = \dfrac{1}{x} + 3$ $\left(= \dfrac{1+3x}{x}\right)$ (C3)

(c) $x \neq 0$ ($\setminus \{0\}$ etc) (A1) (C1)

[6]

4. Discriminant $\Delta = b^2 - 4ac$ $(= (-2k)^2 - 4)$ (A1)

$\Delta > 0$ (M2)

Note: *Award (M1)(M0) for* $\Delta \geq 0$.

$(2k)^2 - 4 > 0 \Rightarrow 4k^2 - 4 > 0$

EITHER

$4k^2 > 4$ $(k^2 > 1)$ (A1)

OR

$4(k-1)(k+1) > 0$ (A1)

OR

$(2k-2)(2k+2) > 0$ (A1)

THEN

$k < -1$ or $k > 1$ (A1)(A1) (C6)

Note: *Award (A1) for* $-1 < k < 1$.

[6]

5. (a) **METHOD 1**

Using the discriminant = 0 ($q^2 - 4(4)(25) = 0$) M1

$q^2 = 400$

$q = 20, q = -20$ A1A1 N2

METHOD 2

Using factorizing:
$(2x - 5)(2x - 5)$ and/or $(2x + 5)(2x + 5)$ M1

$q = 20, q = -20$ A1A1 N2

(b) $x = 2.5$ A1 N1

(c) (0, 25) A1A1 N2

[6]

6. One solution $\Rightarrow$ discriminant = 0 (M2)

$3^2 - 4k = 0$ (A2)

$9 = 4k$

$k = \frac{9}{4}\left(= 2\frac{1}{4}, 2.25\right)$ (A2) (C6)

Note: *If candidates correctly solve an incorrect equation, award M2 A0 A2(ft), if they have the first line or equivalent, otherwise award no marks.*

[6]

7. (a) (i) $m = 3$ A2 N2

(ii) $p = 2$ A2 N2

(b) Appropriate substitution M1

eg $0 = d(1 - 3)^2 + 2$, $0 = d(5 - 3)^2 + 2$, $2 = d(3 - 1)(3 - 5)$

$d = -\frac{1}{2}$ A1 N1

[6]

8. (a) (i) $h = -1$ (A2) (C2)

(ii) $k = 2$ (A1) (C1)

(b) $a(1 + 1)^2 + 2 = 0$ (M1)(A1)

$a = -0.5$ (A1) (C3)

[6]

9. (a) $a = 3, b = 4$ (A1)

$f(x) = (x - 3)^2 + 4$ A1 (C2)

(b) $y = (x - 3)^2 + 4$

METHOD 1

$x = (y - 3)^2 + 4$ (M1)

$x - 4 = (y - 3)^2$

$\sqrt{x - 4} = y - 3$ (M1)

$y = \sqrt{x - 4} + 3$ (A1) 3

METHOD 2

$y - 4 = (x - 3)^2$ (M1)

$\sqrt{y - 4} = x - 3$ (M1)

$\sqrt{y - 4} + 3 = x$

$y = \sqrt{x - 4} + 3$

$\Rightarrow f^{-1}(x) = \sqrt{x - 4} + 3$ (A1) 3

(c) $x \geq 4$ (A1)(C1)

[6]

10. (a) $p = -\frac{1}{2}, q = 2$ (A1)(A1) (C2)

or vice versa

(b) By symmetry C is midway between p, q (M1)

Note: *This (M1) may be gained by implication.*

$\Rightarrow$ x-coordinate is $\frac{-\frac{1}{2} + 2}{2} = \frac{3}{4}$ (A1) (C2)

[4]

11. (a) evidence of attempting to solve $f(x) = 0$ (M1)
evidence of correct working A1

eg $(x+1)(x-2), \dfrac{1 \pm \sqrt{9}}{2}$

intercepts are $(-1, 0)$ and $(2, 0)$ (accept $x = -1$, $x = 2$) A1A1 N1N1

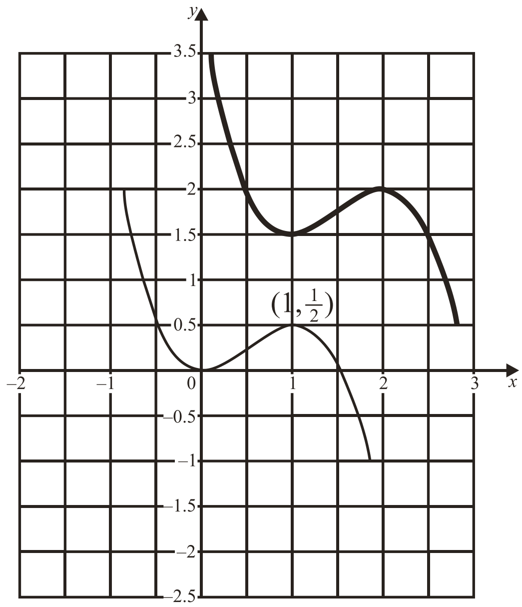
(b) evidence of appropriate method (M1)

eg $x_v = \dfrac{x_1 + x_2}{2}$, $x_v = -\dfrac{b}{2a}$, reference to symmetry

$x_v = 0.5$ A1 N2

[6]

12. (a)



(A2) (C2)

(b) Minimum: $\left(1, \dfrac{3}{2}\right)$ (A1) (C1)

Maximum: (2, 2) (A1) (C1)

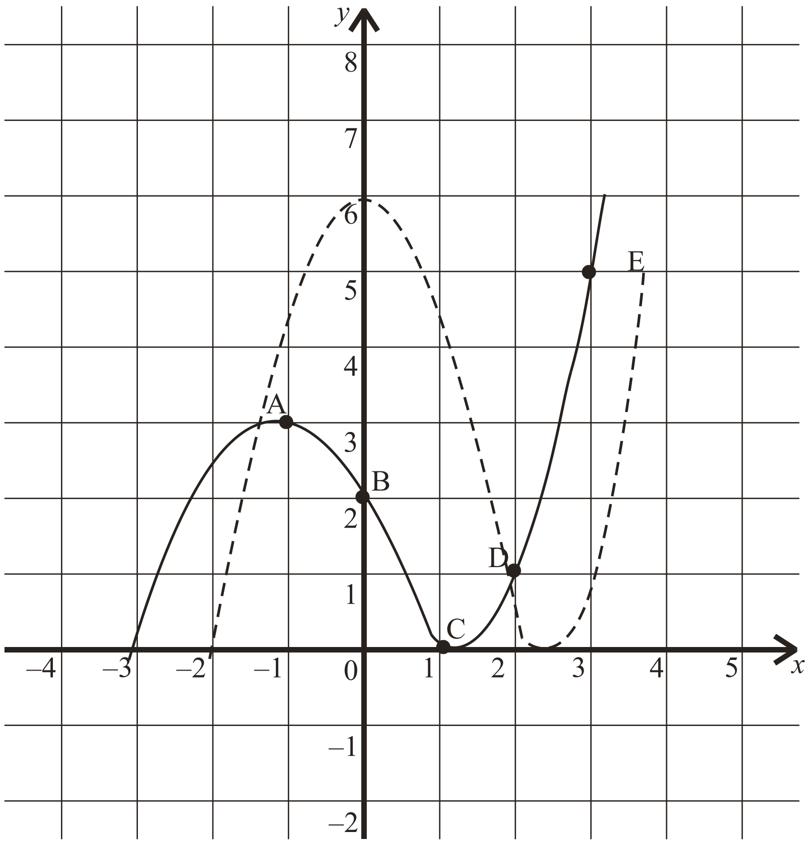
[4]

13. (a) $g(x) = 2f(x - 1)$

x	0	1	2	3
$x - 1$	–1	0	1	2
$f(x - 1)$	3	2	0	1

$g(0) = 2f(-1) = 6$ (A1) (C1)
$g(1) = 2f(0) = 4$ (A1) (C1)
$g(2) = 2f(1) = 0$ (A1) (C1)
$g(3) = 2f(2) = 2$ (A1) (C1)

(b) Graph passing through (0, 6), (1, 4), (2, 0), (3, 2) (A1)
Correct shape. (A1)



(C2)

[6]

14. (a) D A2 N2

(b) C A2 N2

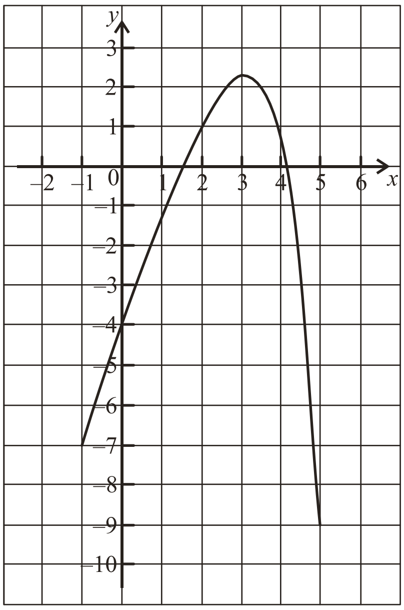
(c) A A2 N2

[6]

15. (a) intercepts when $f(x) = 0$ (M1)

$(1.54, 0)$ $(4.13, 0)$ (accept $x = 1.54$ $x = 4.13$) A1A1 N3

(b)



A1A1A1 N3

Note: *Award A1 for passing through approximately (0, – 4), A1 for correct shape, A1 for a range of approximately −9 to 2.3.*

(c) gradient is 2 A1 N1

[7]

16. (a) For a reasonable attempt to complete the square, (or expanding)

$3x^2 - 12x + 11 = 3(x^2 - 4x + 4) + 11 - 12$

$= 3(x - 2)^2 - 1$ (Accept $h = 2$, $k = 1$) A1A1 2

(b) **METHOD 1**

Vertex shifted to $(2 + 3, -1 + 5) = (5, 4)$ M1

so the new function is $3(x - 5)^2 + 4$ (Accept $p = 5$, $q = 4$) A1A1 2

METHOD 2

$g(x) = 3((x - 3) - h)^2 + k + 5 = 3((x - 3) - 2)^2 - 1 + 5$ M1

$= 3(x - 5)^2 + 4$ (Accept $p = 5$, $q = 4$) A1A1 2

[6]

17. (a) For attempting to complete the square or expanding $y = 2(x - c)^2 + d$, or for showing the vertex is at $(3, 5)$ M1

$y = 2(x - 3)^2 + 5$ (accept $c = 3$, $d = 5$) A1A1 N2

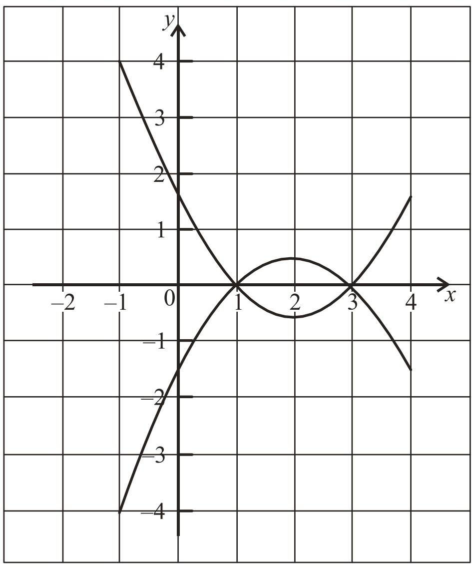
(b) (i) $k = 2$ A1 N1

(ii) $p = 3$ A1 N1

(iii) $q = 5$ A1 N1

[6]

18. (a)



M1A1 N2

Note: *Award M1 for evidence of reflection in x-axis, A1 for correct vertex and all intercepts approximately correct.*

(b) (i) $g(-3) = f(0)$ (A1)

$f(0) = -1.5$ A1 N2

(ii) translation (accept shift, slide, *etc.*) of $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ A1A1 N2

[6]