

Functions 1

1)

(a) $y = \frac{6-x}{2}$

$\Rightarrow x = \frac{6-y}{2}$

(M1)

$\Rightarrow y = 6 - 2x = g^{-1}(x)$

(A1)

(C2)

(b) $(f \circ g^{-1})(x) = 4[(6 - 2x) - 1] = 4(5 - 2x) = 20 - 8x$

(M1)(A1)

$20 - 8x = 4 \Rightarrow 8x = 16$

(M1)

$\Rightarrow x = 2$

(A1)

(C4)

[6 marks]

2)

QUESTION 12

(a) $a = 3, b = 4$

(A1)

$f(x) = (x-3)^2 + 4$

A1

C2

(b) $y = (x-3)^2 + 4$

METHOD 1

$x = (y-3)^2 + 4$

(M1)

$x - 4 = (y-3)^2$

$\sqrt{x-4} = y-3$

(M1)

$y = \sqrt{x-4} + 3$

A1

C3

METHOD 2

$y - 4 = (x-3)^2$

(M1)

$\sqrt{y-4} = x-3$

(M1)

$\sqrt{y-4} + 3 = x$

$y = \sqrt{x-4} + 3$

$\Rightarrow f^{-1}(x) = \sqrt{x-4} + 3$

A1

C3

(c) $x \geq 4$

A1

C1

3)

(a) $x = \frac{8}{y}$

(M1)

$y = \frac{8}{x}$

(A1)

(C2)

(b) (i) $(f^{-1} \circ g)(x) = \frac{8}{x^2}$

(A2)

(C2)

(ii) $\frac{8}{x^2} = x$

(M1)

$x = 2$

(A1)

(C2)

Functions 1

- 4)
- (a) $y = 2x + 1$
 $x = 2y + 1$ *(M1)*
 $\frac{x-1}{2} = y$
 $f^{-1}(x) = \frac{x-1}{2}$ *(A1)* *(C2)*
- (b) $g(f(-2)) = g(-3)$ *(A1)*
 $= 3(-3)^2 - 4$
 $= 23$ *(A1)* *(C2)*
- (c) $f(g(x)) = f(3x^2 - 4)$
 $= 2(3x^2 - 4) + 1$ *(A1)*
 $= 6x^2 - 7$ *(A1)* *(C2)*
- 5)
- (a) **METHOD 1**
 $(f \circ g)(4) = f(g(4)) = f(1)$ *(M1)*
 $= 2$ *(A1)* *(C2)*
- METHOD 2**
 $(f \circ g)(x) = \frac{2}{x-3}$ *(M1)*
 $(f \circ g)(4) = 2$ *(A1)* *(C2)*
- (b) Let $y = \frac{1}{x-3}$
 Correct simplification $y(x-3) = 1$ $\left(x-3 = \frac{1}{y}\right)$ *(A1)*
 $x = \frac{1}{y} + 3$ $\left(= \frac{1+3y}{y}\right)$ *(A1)*
 Interchanging x and y (may happen earlier) *(M1)*
 $y = \frac{1}{x} + 3$ $\left(= \frac{1+3x}{x}\right)$ *(C3)*
- (c) $x \neq 0$ ($\mathbb{R} \setminus \{0\}$ etc) *(A1)* *(C1)*
- 6)
- (a) D *A2* *N2*
- (b) C *A2* *N2*
- (c) A *A2* *N2*

Functions 1

- 7) (a) **METHOD 1**
- For $f(-2) = -12$ *(A1)*
 $(g \circ f)(-2) = g(-12) = -24$ *A1* *N2*
- METHOD 2**
- $(g \circ f)(x) = 2x^3 - 8$ *(A1)*
 $(g \circ f)(-2) = -24$ *A1* *N2*
- (b) Interchanging x and y (may be done later) *(M1)*
 $x = y^3 - 4$ *A1*
 $f^{-1}(x) = \sqrt[3]{x+4}$ *A2* *N3*
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- 8) (a) **METHOD 1**
- Attempting to interchange x and y *(M1)*
Correct expression $x = 3y - 5$ *(A1)*
 $f^{-1}(x) = \frac{x+5}{3}$ *A1* *N3*
- METHOD 2**
- Attempting to solve for x in terms of y *(M1)*
Correct expression $x = \frac{y+5}{3}$ *(A1)*
 $f^{-1}(x) = \frac{x+5}{3}$ *A1* *N3*
- [3 marks]*
- (b) For correct composition $(g^{-1} \circ f)(x) = (3x-5)+2$ *(A1)*
 $(g^{-1} \circ f)(x) = 3x-3$ *A1* *N2*
- [2 marks]*
- (c) $\frac{x+3}{3} = 3x-3$ ($x+3 = 9x-9$) *(A1)*
 $x = \frac{12}{8}$ *A1* *N2*
- [2 marks]*