Ch 9 Functions Test 2

5. i. minimum when $x = \frac{1}{3} \Rightarrow f\left(\frac{1}{3}\right) = -11$

greatest value is $f(-3) = 89$

3 $\therefore \underline{\underline{-11 \leq f(x) \leq 89}}$

ii. a. minimum at $\left(\frac{1}{3}, -11\right)$

4 b. maximum at $\left(\frac{1}{3}, 11\right)$

6. 1 i. gf

1 ii. $g^{-1}f$

1 iii. $f^{-1}g$

7. i. Let $y = 3x - 2$

$\therefore 3x = y + 2$

$\therefore x = \frac{y+2}{3}$

$\therefore \underline{\underline{f^{-1} = \frac{x+2}{3}}}$

3

Let $y = \frac{7x - a}{x + 1}$

$\therefore xy + y = 7x - a$

$\therefore x(y - 7) = -a - y$

$\therefore x = \frac{a + y}{7 - y}$

$\therefore \underline{\underline{g^{-1} = \frac{a + x}{7 - x}}}$

ii. Require $f^{-1}\left(\frac{28 - a}{5}\right) = 2$

$\therefore \frac{\frac{28-a}{5} + 2}{3} = 2$

$\therefore \frac{28 - a}{5} + 2 = 6$

$\therefore 28 - a = 20$

3 $\therefore \underline{\underline{a = 8}}$

7 iii. $g(x) = g^{-1}(x)$

$\therefore \dfrac{7x-9}{x+1} = \dfrac{9+x}{7-x}$

$\therefore (7x-9)(7-x) = (9+x)(x+1)$

$\therefore 49x - 7x^2 - 63 + 9x = 9x + 9 + x^2 + x$

$\therefore 8x^2 - 48x + 72 = 0$

$\therefore x^2 - 6x + 9 = 0$

$\therefore (x-3)^2 = 0$

3 $\therefore x = 3$ $\therefore$ only one value of x, as required.

8. i. range $= \mathbb{R}$ or $\underline{\underline{-\infty < f < \infty}}$

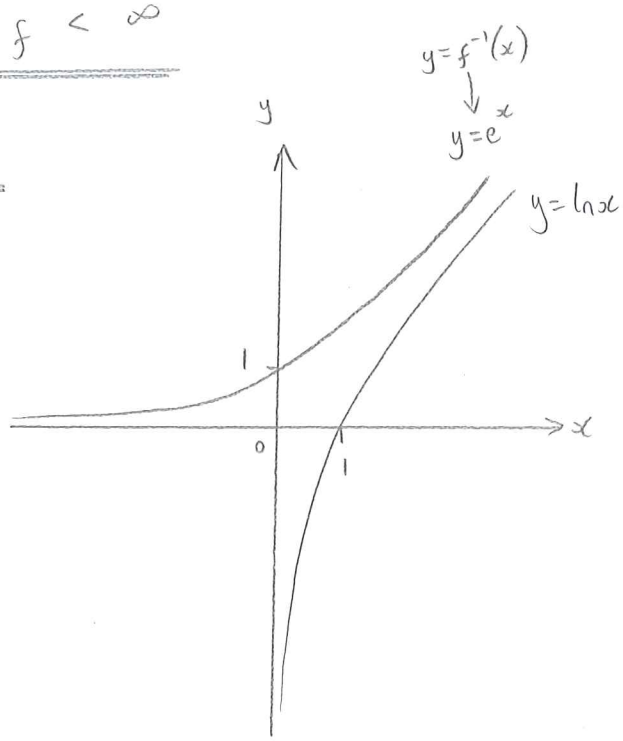
1

ii. $f^{-1}(x) = e^x$ $\therefore$ $\underline{\underline{f^{-1} > 0}}$

1

iii. graphs →

2



iv. $fg(x) = 3$

$\therefore f(3x+2) = 3$

$\therefore \ln(3x+2) = 3$

$\therefore 3x + 2 = e^3$

$\therefore 3x = e^3 - 2$

2 $\therefore \underline{\underline{x = \dfrac{e^3-2}{3}}}$ $(= 6.03)$

v. $f^{-1}g^{-1} = 7 \Rightarrow g^{-1} = f(7) \Rightarrow x = gf(7)$

$= g(\ln 7)$

4 $= \underline{\underline{3\ln 7 + 2}}$

$(= 7.84)$