

Functions 2

- 1) Solve the equation $|4x - 5| = 21$. [3]
- 2) The functions f and g are defined, for $x \in \mathbb{R}$, by
- $$f : x \mapsto 3x - 2,$$
- $$g : x \mapsto \frac{7x - a}{x + 1}, \text{ where } x \neq -1 \text{ and } a \text{ is a positive constant.}$$
- (i) Obtain expressions for f^{-1} and g^{-1} . [3]
- (ii) Determine the value of a for which $f^{-1}g(4) = 2$. [3]
- (iii) If $a = 9$, show that there is only one value of x for which $g(x) = g^{-1}(x)$. [3]
- 3) (i) In the space below sketch the graph of $y = |(2x + 3)(2x - 7)|$. [4]
- (ii) How many values of x satisfy the equation $|(2x + 3)(2x - 7)| = 2x$? [2]
- 4) (i) Sketch the graph of $y = |3x - 5|$, for $-2 \leq x \leq 3$, showing the coordinates of the points where the graph meets the axes. [3]
- (ii) On the same diagram, sketch the graph of $y = 8x$. [1]
- (iii) Solve the equation $8x = |3x - 5|$. [3]
- 5) Solve the equation $|2x + 10| = 7$. [3]
- 6) The function f is defined, for $x > 0$, by $f : x \mapsto \ln x$.
- (i) State the range of f . [1]
- (ii) State the range of f^{-1} . [1]
- (iii) On the same diagram, sketch and label the graphs of $y = f(x)$ and $y = f^{-1}(x)$. [2]
- The function g is defined, for $x > 0$, by $g : x \mapsto 3x + 2$.
- (iv) Solve the equation $fg(x) = 3$. [2]
- (v) Solve the equation $f^{-1}g^{-1}(x) = 7$. [4]

7)

(a) Functions f and g are defined, for $x \in \mathbb{R}$, by

$$f(x) = 3 - x,$$

$$g(x) = \frac{x}{x+2}, \text{ where } x \neq -2.$$

(i) Find $fg(x)$. [2]

(ii) Hence find the value of x for which $fg(x) = 10$. [2]

(b) A function h is defined, for $x \in \mathbb{R}$, by $h(x) = 4 + \ln x$, where $x > 1$.

(i) Find the range of h . [1]

(ii) Find the value of $h^{-1}(9)$. [2]

(iii) On the same axes, sketch the graphs of $y = h(x)$ and $y = h^{-1}(x)$. [3]