Functions 1

1) **(a)** The function f is such that $f(x) = 2x^2 - 8x + 5$.

(i) Show that $f(x) = 2(x + a)^2 + b$, where a and b are to be found. [2]

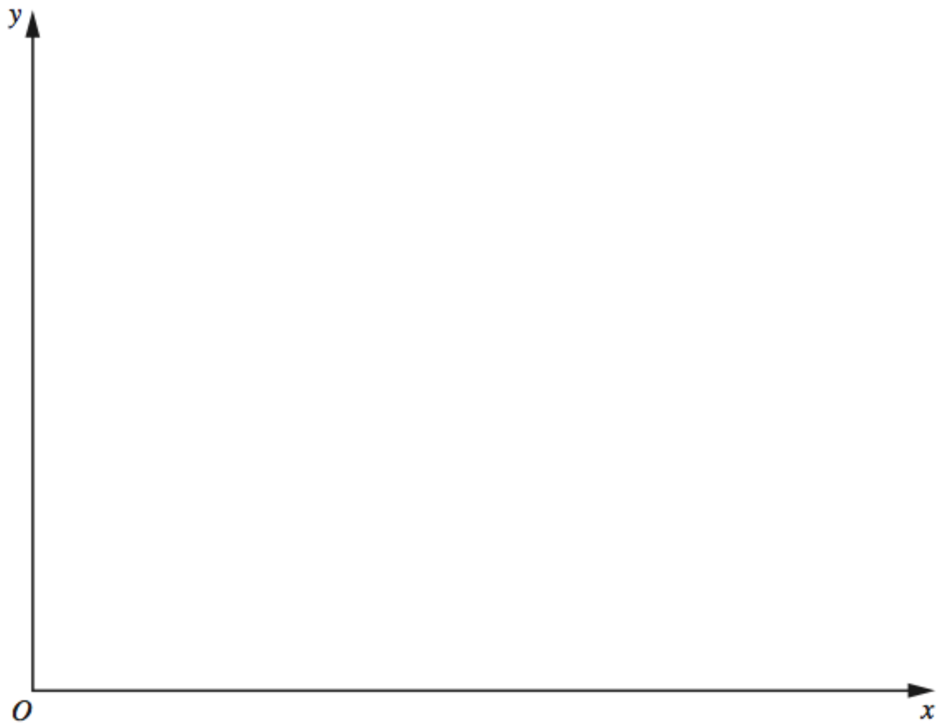
(ii) Hence, or otherwise, write down a suitable domain for f so that f^{-1} exists. [1]

(b) The functions g and h are defined respectively by

$$g(x) = x^2 + 4, \quad x \geqslant 0, \quad h(x) = 4x - 25, x \geqslant 0.$$

(i) Write down the range of g and of h^{-1}. [2]

(ii) On the axes below, sketch the graphs of $y = g(x)$ and $y = g^{-1}(x)$, showing the coordinates of any points where the curves meet the coordinate axes. [3]



(iii) Find the value of x for which $gh(x) = 85$. [4]

2) A function g is defined by $g : x \to (x + 3)^2 - 7$ for $x > -3$.

(i) Find an expression for $g^{-1}(x)$. [2]

(ii) Solve the equation $g^{-1}(x) = g(0)$. [3]

3) The function f is defined by

$$f(x) = (2x + 1)^2 - 3 \quad \text{for } x \geqslant -\frac{1}{2}.$$

Find

(i) the range of f, [1]

(ii) an expression for $f^{-1}(x)$. [3]

The function g is defined by

$$g(x) = \frac{3}{1 + x} \quad \text{for } x > -1.$$

(iii) Find the value of x for which $fg(x) = 13$. [4]

4) **(i)** Sketch the graph of $y = |3x + 9|$ for $-5 < x < 2$, showing the coordinates of the points where the graph meets the axes. [3]

(ii) On the same diagram, sketch the graph of $y = x + 6$. [1]

(iii) Solve the equation $|3x + 9| = x + 6$. [3]

5) Sketch the graph of $y = |x^2 - 8x + 12|$. [4]

6) **(i)** Sketch, on the same diagram, the graphs of $y = x - 3$ and $y = |2x - 9|$. [3]

(ii) Solve the equation $|2x - 9| = x - 3$. [2]