EXPECTED AND BINOMIAL 1

1) The probability distribution of the discrete random variable X is given by the following table.

x	1	2	3	4	5
$\mathrm{P}(X = x)$	0.4	p	0.2	0.07	0.02

(a) Find the value of p.

(b) Calculate the expected value of X.

2) Three students, Kim, Ching Li and Jonathan each have a pack of cards, from which they select a card at random. Each card has a 0, 3, 4, or 9 printed on it.

(a) Kim states that the probability distribution for her pack of cards is as follows.

x	0	3	4	9
$\mathrm{P}(X = x)$	0.3	0.45	0.2	0.35

Explain why Kim is incorrect. *[2 marks]*

(b) Ching Li correctly states that the probability distribution for her pack of cards is as follows.

x	0	3	4	9
$\mathrm{P}(X = x)$	0.4	k	$2k$	0.3

Find the value of k. *[2 marks]*

(c) Jonathan correctly states that the probability distribution for his pack of cards is given by $\mathrm{P}(X = x) = \dfrac{x+1}{20}$. One card is drawn at random from his pack.

(i) Calculate the probability that the number on the card drawn is 0.

(ii) Calculate the probability that the number on the card drawn is greater than 0. *[4 marks]*

3) *[Maximum mark: 16]*

A **four-sided** die has three blue faces and one red face. The die is rolled.

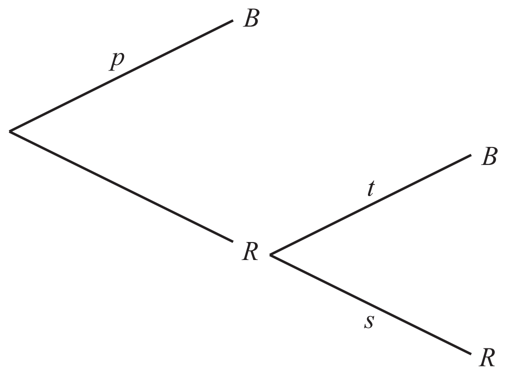
Let B be the event a blue face lands down, and R be the event a red face lands down.

(a) Write down

(i) $\mathrm{P}(B)$;

(ii) $\mathrm{P}(R)$. *[2 marks]*

(b) If the blue face lands down, the die is not rolled again. If the red face lands down, the die is rolled once again. This is represented by the following tree diagram, where p, s, t are probabilities.



Find the value of p, of s and of t. *[2 marks]*

Guiseppi plays a game where he rolls the die. If a blue face lands down, he scores 2 and is finished. If the red face lands down, he scores 1 and rolls one more time. Let X be the total score obtained.

(c) (i) Show that $\mathrm{P}(X = 3) = \dfrac{3}{16}$.

(ii) Find $\mathrm{P}(X = 2)$. *[3 marks]*

(d) (i) Construct a probability distribution table for X.

(ii) Calculate the expected value of X. *[5 marks]*

(e) If the total score is 3, Guiseppi wins \$ 10. If the total score is 2, Guiseppi gets nothing.

Guiseppi plays the game twice. Find the probability that he wins exactly \$ 10. *[4 marks]*

4) *[Maximum mark: 7]*

A factory makes switches. The probability that a switch is defective is 0.04. The factory tests a random sample of 100 switches.

(a) Find the mean number of defective switches in the sample. *[2 marks]*

(b) Find the probability that there are exactly six defective switches in the sample. *[2 marks]*

(c) Find the probability that there is at least one defective switch in the sample. *[3 marks]*

5) The following table shows the probability distribution of a discrete random variable X.

x	-1	0	2	3
$\mathrm{P}(X = x)$	0.2	$10k^2$	0.4	$3k$

(a) Find the value of k. *[4 marks]*

(b) Find the expected value of X. *[3 marks]*

6) A box holds 240 eggs. The probability that an egg is brown is 0.05.

(a) Find the expected number of brown eggs in the box. *[2 marks]*

(b) Find the probability that there are 15 brown eggs in the box. *[2 marks]*

(c) Find the probability that there are at least 10 brown eggs in the box. *[3 marks]*

7) The random variable X has the following probability distribution.

x	1	2	3
$\mathrm{P}(X = x)$	s	0.3	q

Given that $\mathrm{E}(X) = 1.7$, find q.

8) A company produces a large number of water containers. Each container has two parts, a bottle and a cap. The bottles and caps are tested to check that they are not defective.

A cap has a probability of 0.012 of being defective. A random sample of 10 caps is selected for inspection.

(a) Find the probability that exactly one cap in the sample will be defective. *[2 marks]*

(b) The sample of caps passes inspection if at most one cap is defective. Find the probability that the sample passes inspection. *[2 marks]*