End of year revision

163 min
169 marks

1. A woman deposits $100 into her son's savings account on his first birthday. On his second birthday she deposits $125, $150 on his third birthday, and so on.

(a) How much money would she deposit into her son's account on his 17th birthday?

(b) How much in total would she have deposited after her son's 17th birthday?

(Total 4 marks)

2. The first four terms of an arithmetic sequence are shown below.

1, 5, 9, 13,......

(a) Write down the nth term of the sequence.

(b) Calculate the 100th term of the sequence.

(c) Find the sum of the first 100 terms of the sequence.

(Total 4 marks)

3. Consider the following sequence:

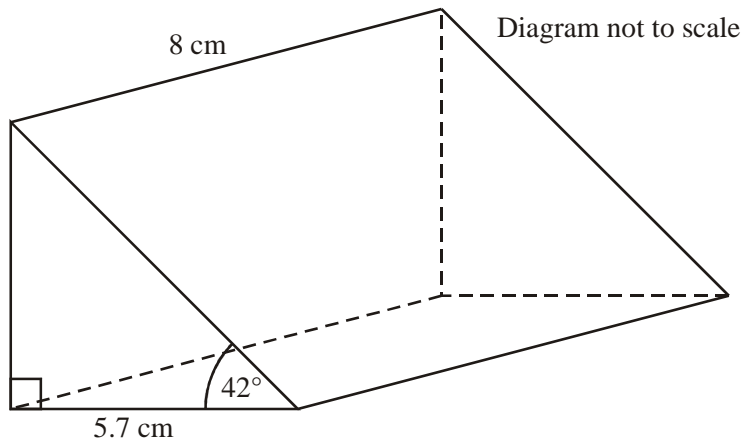$$57, 55, 53 \ldots , 5, 3$$

(a) Find the number of terms of the sequence.

(3)

(b) Find the sum of the sequence.

(3)

(Total 6 marks)

4. Find the volume of the following prism.

8 cm

Diagram not to scale

42°

5.7 cm

(Total 4 marks)

5. The right pyramid shown in the diagram has a square base with sides of length 40 cm. The height of the pyramid is also 40 cm.
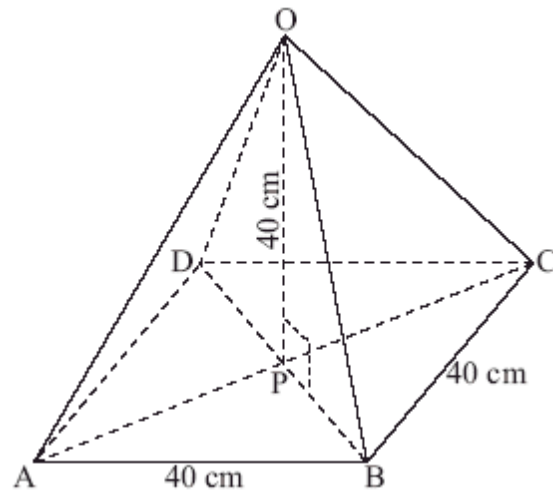

diagram not to scale

(a) Find the length of OB.

(4)

(b) Find the size of angle OB̂P .

(2)

(Total 6 marks)

6. The diagram shows a pyramid VABCD which has a square base of length 10 cm and edges of length 13 cm. M is the midpoint of the side BC.
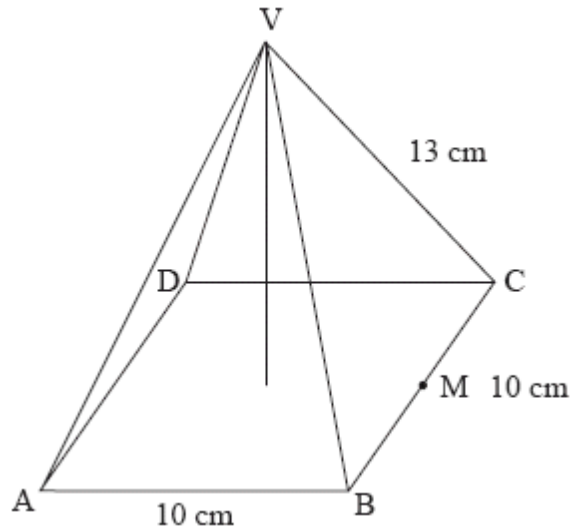

diagram not to scale

(a) Calculate the length of VM.

(2)

(b) Calculate the vertical height of the pyramid.

(2)

(c) Calculate the angle between a sloping face of the pyramid and its base.

(2)

(Total 6 marks)

7. The marks obtained by 8 candidates in Physics and Chemistry tests are given below.

Physics (x)	6	8	10	11	10	5	4	12
Chemistry (y)	8	11	14	13	11	7	5	15

(a) Write down the product moment correlation coefficient, r.

(1)

(b) Write down, in the form $y = mx + c$, the equation of the regression line y on x for the 8 candidates.

(2)

A ninth candidate obtained a score of 7 in the Physics test but was absent for the Chemistry test.

(c) Use your answer to (b) to estimate the score he would have obtained on the Chemistry test.

(2)

(d) Give a reason why it is valid to use this regression line to estimate the score on the Chemistry test.

(1)

(Total 6 marks)

8. The eye colour and gender of 500 students are noted and the results are indicated in the table below.

	Blue	Brown	Green
Male	18	152	50
Female	40	180	60

It is believed that eye colour is related to gender in a school in Banff. It is decided to test this hypothesis by using a χ^2 test at the 5% level of significance.

(a) Write down the null hypothesis for this experiment.

(1)

(b) Show that the number of degrees of freedom is 2.

(1)

(c) Write down the χ^2 critical value for the degrees of freedom.

(1)

(d)   Calculate the χ^2 test statistic for this data.

(2)

(e)   Does the evidence suggest that eye colour is related to gender in this school? Give a clear reason for your answer.

(2)
(Total 7 marks)

9.   200 people of different ages were asked to choose their favourite type of music from the choices Popular, Country and Western and Heavy Metal. The results are shown in the table below.

Age/Music choice	Popular	Country and Western	Heavy Metal	Totals
11–25	35	5	50	90
26–40	30	10	20	60
41–60	20	25	5	50
Totals	85	40	75	200

It was decided to perform a chi-squared test for independence at the 5% level on the data.

(a)   Write down the null hypothesis.

(1)

(b)   Write down the number of degrees of freedom.

(1)

(c)   Write down the chi-squared value.

(2)

(d)   State whether or not you will reject the null hypothesis, giving a clear reason for your answer.

(2)
(Total 6 marks)

10. The temperatures in °C, at midday in Geneva, were measured for eight days and the results are recorded below.

$$7, 4, 5, 4, 8, T, 14, 4$$

The mean temperature was found to be 7 °C.

(a) Find the value of T.

(3)

(b) Write down the mode.

(1)

(c) Find the median.

(2)
(Total 6 marks)

11. A line joins the points A(2, 1) and B(4, 5).

(a) Find the gradient of the line AB.

(2)

Let M be the midpoint of the line segment AB.

(b) Write down the coordinates of M.

(1)

(c) Find the equation of the line perpendicular to AB and passing through M.

(3)
(Total 6 marks)

12. The weights in kg, of 80 adult males, were collected and are summarized in the box and whisker plot shown below.
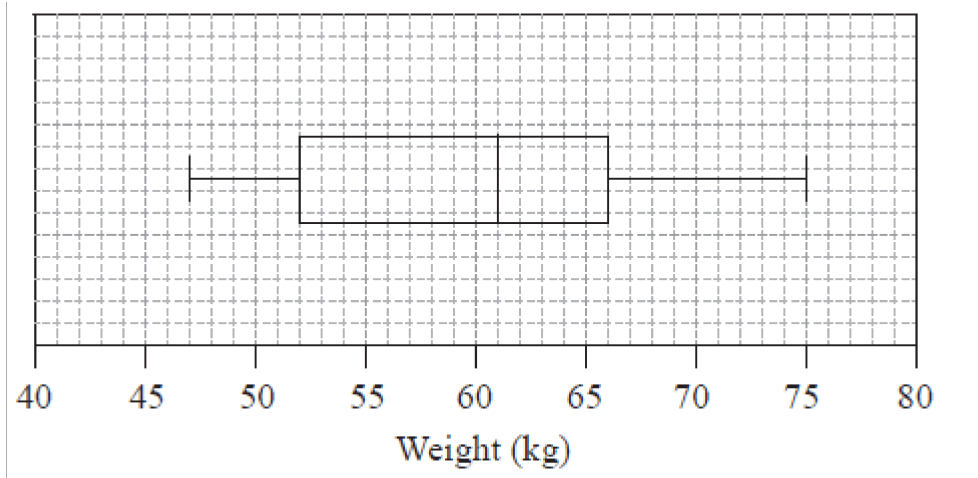
(a) Write down the median weight of the males.

(1)

(b) Calculate the interquartile range.

(2)

(c) Estimate the number of males who weigh between 61 kg and 66 kg.

(1)

(d) Estimate the mean weight of the lightest 40 males.

(2)

(Total 6 marks)

13. The straight line, L_1, has equation $y = -\dfrac{1}{2}x - 2$.

(a) Write down the y intercept of L_1.

(1)

(b) Write down the gradient of L_1.

(1)

The line L_2 is perpendicular to L_1 and passes through the point (3, 7).

(c)   Write down the gradient of the line L_2.

(1)

(d)   Find the equation of L_2. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

(3)
(Total 6 marks)

14. The straight line L passes through the points A(–1, 4) and B(5, 8).

(a)   Calculate the gradient of L.

(2)

(b)   Find the equation of L.

(2)

The line L also passes through the point P(8, y).

(c)   Find the value of y.

(2)
(Total 6 marks)

15. The heat output in thermal units from burning 1 kg of wood changes according to the wood's percentage moisture content. The moisture content and heat output of 10 blocks of the same type of wood each weighing 1 kg were measured. These are shown in the table.

Moisture content % (x)	8	15	22	30	34	45	50	60	74	82
Heat output (y)	80	77	74	69	68	61	61	55	50	45

(a)   Draw a scatter diagram to show the above data. Use a scale of 2 cm to represent 10 % on the x-axis and a scale of 2 cm to represent 10 thermal units on the y-axis.

(4)

(b)　Write down

　　(i)　the mean percentage moisture content, $\bar{x}$;

　　(ii)　the mean heat output, $\bar{y}$.

(2)

(c)　Plot the point $(\bar{x}, \bar{y})$ on your scatter diagram and label this point M.

(2)

(d)　Write down the product-moment correlation coefficient, *r*.

(2)

The equation of the regression line *y* on *x* is $y = -0.470x + 83.7$.

(e)　Draw the regression line *y* on *x* on your scatter diagram.

(2)

(f)　Estimate the heat output in thermal units of a 1 kg block of wood that has 25 % moisture content.

(2)

(g)　State, with a reason, whether it is appropriate to use the regression line *y* on *x* to estimate the heat output in part (f).

(2)
(Total 16 marks)

16.　80 matches were played in a football tournament. The following table shows the number of goals scored in all matches.

Number of goals	0	1	2	3	4	5
Number of matches	16	22	19	17	1	5

(a)　Find the mean number of goals scored per match.

(2)

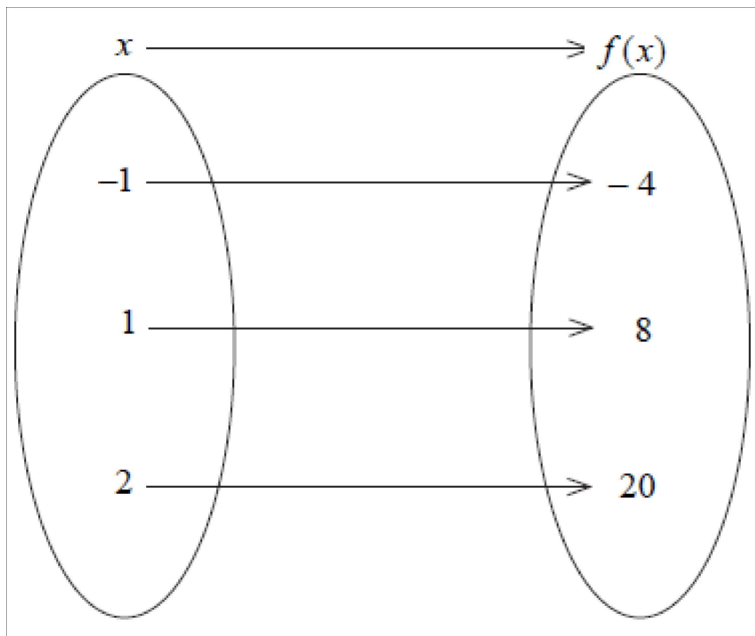(b)   Find the median number of goals scored per match.

(2)

A local newspaper claims that the mean number of goals scored per match is two.

(c)   Calculate the percentage error in the local newspaper's claim.

(2)

(Total 6 marks)

17.   A quadratic function, $f(x) = ax^2 + bx$, is represented by the mapping diagram below.

(a)   Use the mapping diagram to write down **two** equations in terms of a and b.

(2)

(b)   Find the value of

(i) a;

(ii) b.

(2)

(c)   Calculate the *x*-coordinate of the vertex of the graph of *f*(*x*).

(2)
(Total 6 marks)

18.   Let $f(x) = x^2 - 6x + 8$.

(a)   Factorise $x^2 - 6x + 8$.

(2)

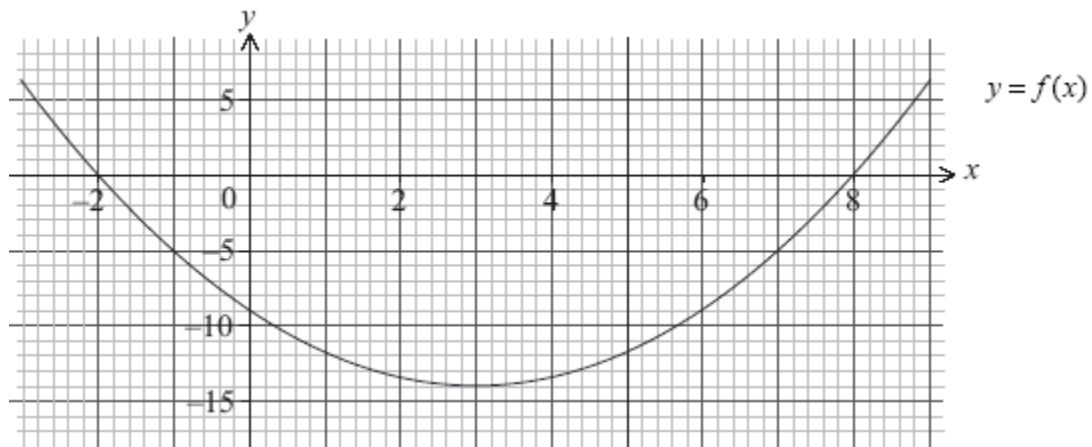(b)   Hence, or otherwise, solve the equation $x^2 - 6x + 8 = 0$.

(2)

Let $g(x) = x + 3$.

(c)   Write down the solutions to the equation $f(x) = g(x)$.

(2)
(Total 6 marks)

19.   The graph of a quadratic function $y = f(x)$ is given below.

(a) Write down the equation of the axis of symmetry.

(2)

(b) Write down the coordinates of the minimum point.

(2)

(c) Write down the range of $f(x)$.

(2)

(Total 6 marks)

20. (a) List the elements of the set $A = \{x \mid -4 \leq x \leq 2, x \text{ is an integer}\}$.

(1)

A number is chosen at random from set A.

Write down the probability that the number chosen is

(b) a negative integer;

(2)

(c) a positive even integer;

(1)

(d) an odd integer less than –1.

(2)

(Total 6 marks)

21. A problem has an **exact** answer of $x = 0.1265$.

(a) Write down the **exact** value of x in the form $a \times 10^k$ where k is an integer and $1 \leq a \leq 10$.

(b) State the value of x given correct to **two** significant figures.

(c) Calculate the percentage error if x is given correct to **two** significant figures.

(Total 6 marks)

22. The total weight of 256 identical pencils is 4.24 kg. Calculate the weight of one pencil, in kg.

    (a)   Give your answer exactly.

    (b)   Give your answer correct to three significant figures.

    (c)   Write your answer to part (b) in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(Total 8 marks)

23. The diagram shows triangle ABC in which angle $\hat{BAC} = 30°$, BC = 6.7 cm and AC = 13.4 cm.
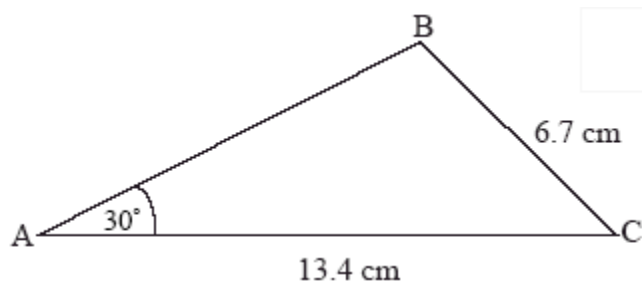

diagram not to scale

    (a)   Calculate the size of angle $\hat{ACB}$.

(4)

Nadia makes an accurate drawing of triangle ABC. She measures angle $\hat{BAC}$ and finds it to be 29°.

    (b)   Calculate the percentage error in Nadia's measurement of angle $\hat{BAC}$.

(2)
(Total 6 marks)

24.   A farmer has a triangular field, ABC, as shown in the diagram.
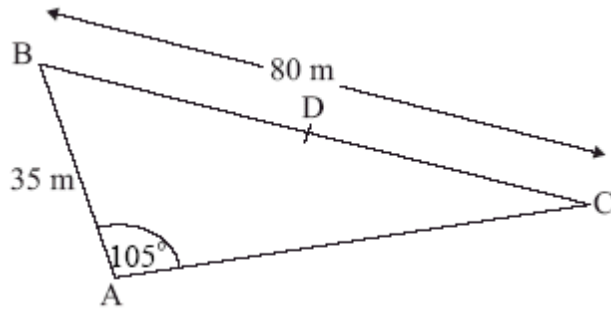AB = 35 m, BC = 80 m and BÂC =105°, and D is the midpoint of BC.

diagram not to scale

(a)   Find the size of BĈA.

(3)

(b)   Calculate the length of AD.

(5)

The farmer wants to build a fence around ABD.

(c)   Calculate the total length of the fence.

(2)

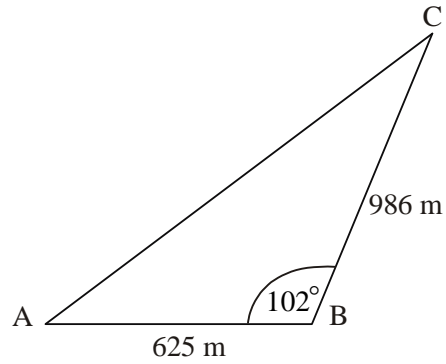(d)   The farmer pays 802.50 USD for the fence. Find the cost per metre.

(2)

(e)   Calculate the area of the triangle ABD.

(3)

(f)   A layer of earth 3 cm thick is removed from ABD. Find the volume removed in cubic
metres.

(3)
(Total 18 marks)

25. On a map three schools A, B and C are situated as shown in the diagram.

Schools A and B are 625 metres apart.
Angle $A\hat{B}C = 102°$ and BC = 986 metres.

(a) Find the distance between A and C.

(3)

(b) Find the size of angle $B\hat{A}C$.

(3)
(Total 6 marks)