Differentiation Practice Questions

A. Chain, product and quotient rule

1. Differentiate with respect to x

(a) $\sqrt{3-4x}$

(b) $e^{\sin x}$

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

2. Differentiate with respect to x:

(a) $(x^2 + 1)^2$.

(b) $\ln(3x - 1)$.

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

3. Let $f(x) = e^{\frac{x}{3}} + 5\cos^2 x$. Find $f'(x)$.

Working:

Answer:

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(Total 6 marks)

4. Let $f(x) = 6\sqrt[3]{x^2}$. Find $f'(x)$.

Working:

Answer:

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(Total 6 marks)

5. Differentiate each of the following with respect to x.

(a) $y = \sin 3x$ **(1)**

(b) $y = x \tan x$ **(2)**

(c) $y = \dfrac{\ln x}{x}$ **(3)**

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(Total 6 marks)

6. Let $f(x) = \dfrac{3x^2}{5x-1}$.

(a) Write down the **equation** of the vertical asymptote of $y = f(x)$.

(1)

(b) Find $f'(x)$. Give your answer in the form $\dfrac{ax^2+bx}{(5x-1)^2}$ where a and $b \in \mathbb{Z}$.

(4)

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(Total 5 marks)

7. (a) Let $f(x) = e^{5x}$. Write down $f'(x)$.

(b) Let $g(x) = \sin 2x$. Write down $g'(x)$.

(c) Let $h(x) = e^{5x}\sin 2x$. Find $h'(x)$.

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(Total 6 marks)

B. Gradients at particular points and rate of change

8. Let $f(x) = x^3 - 2x^2 - 1$.

(a) Find $f'(x)$.

(b) Find the gradient of the curve of $f(x)$ at the point $(2, -1)$.

Working:

Answers:

(a) …………………………………………

(b) …………………………………………

(Total 6 marks)

9. The population p of bacteria at time t is given by $p = 100e^{0.05t}$. Calculate:

(a) the value of p when $t = 0$;

(b) the rate of increase of the population when $t = 10$.

Working:

Answers:

(a) …………………………………………..

(b) …………………………………………..

(Total 6 marks)

10. Given the function $f(x) = x^2 - 3bx + (c + 2)$, determine the values of b and c such that $f(1) = 0$ and $f'(3) = 0$.

Working:

Answer:

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(Total 4 marks)

C. First Principles

11. Let $f(x) = x^3$.

(a) Evaluate $\dfrac{f(5+h) - f(5)}{h}$ for $h = 0.1$.

(b) What number does $\dfrac{f(5+h) - f(5)}{h}$ approach as h approaches zero?

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

D. Asymptotes, turning points and points of inflexion

12. Let the function f be defined by $f(x) = \dfrac{2}{1+x^3}$, $x \neq -1$.

(a) (i) Write down the equation of the vertical asymptote of the graph of f.

(ii) Write down the equation of the horizontal asymptote of the graph of f.

(iii) Sketch the graph of f in the domain $-3 \leq x \leq 3$.

(4)

(b) (i) Using the fact that $f'(x) = \dfrac{-6x^2}{(1+x^3)^2}$, show that the second derivative $f''(x) = \dfrac{12x(2x^3 - 1)}{(1+x^3)^3}$.

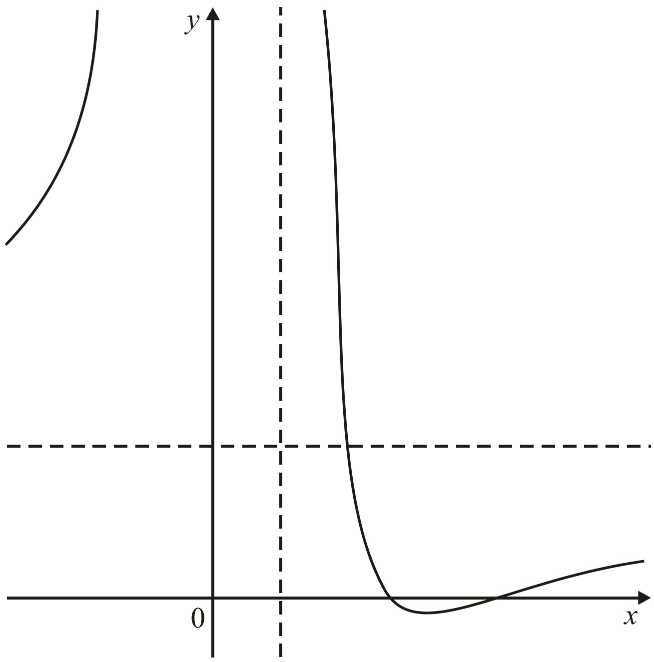
(ii) Find the x-coordinates of the points of inflexion of the graph of f.

(6)

(Total 10 marks)

13. Consider the function f given by $f(x) = \dfrac{2x^2 - 13x + 20}{(x-1)^2}$, $\quad x \neq 1$.

A part of the graph of f is given below.



The graph has a vertical asymptote and a horizontal asymptote, as shown.

(a) Write down the **equation** of the vertical asymptote.

(1)

(b) $f(100) = 1.91 \quad f(-100) = 2.09 \quad f(1000) = 1.99$

(i) Evaluate $f(-1000)$.

(ii) Write down the **equation** of the horizontal asymptote.

(2)

(c) Show that $f'(x) = \dfrac{9x-27}{(x-1)^3}$, $\quad x \neq 1$.

(3)

The second derivative is given by $f''(x) = \dfrac{72-18x}{(x-1)^4}$, $\quad x \neq 1$.

(d) Using values of $f'(x)$ and $f''(x)$ explain why a minimum must occur at $x = 3$.

(2)

(e) There is a point of inflexion on the graph of f. Write down the coordinates of this point.

(2)

(Total 10 marks)

14. Let $f(x) = \dfrac{1}{1+x^2}$.

(a) Write down the equation of the horizontal asymptote of the graph of f.

(1)

(b) Find $f'(x)$.

(3)

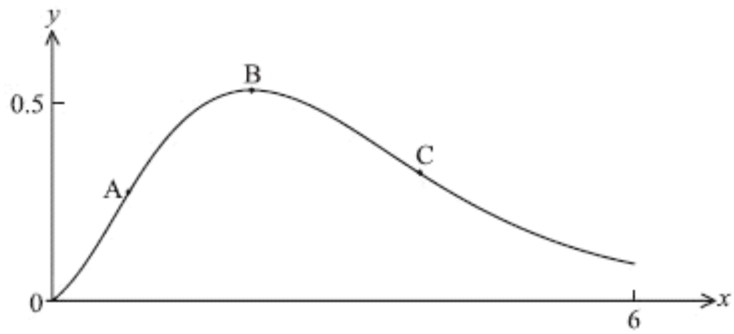
(c) The second derivative is given by $f''(x) = \dfrac{6x^2 - 2}{(1+x^2)^3}$.

Let A be the point on the curve of f where the gradient of the tangent is a maximum. Find the x-coordinate of A.

(4)

(Total 8 marks)

15. The diagram below shows the graph of $f(x) = x^2 e^{-x}$ for $0 \le x \le 6$. There are points of inflexion at A and C and there is a maximum at B.



(a) Using the product rule for differentiation, find $f'(x)$.

(b) Find the **exact** value of the ***y*-coordinate** of B.

(c) The second derivative of f is $f''(x) = (x^2 - 4x + 2)\, e^{-x}$. Use this result to find the **exact** value of the x-coordinate of C.

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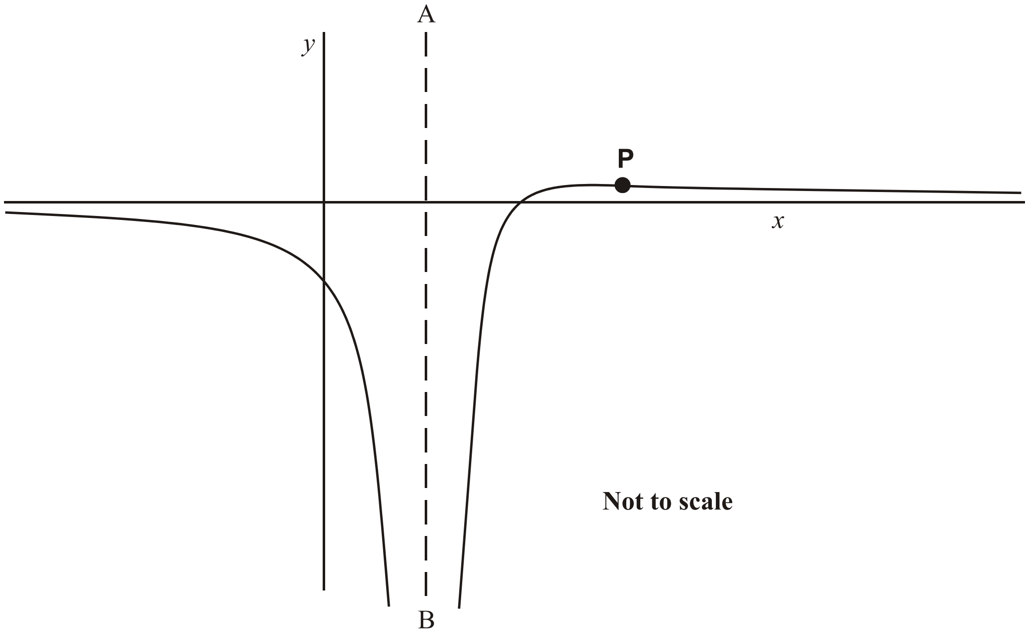
(Total 6 marks)

16. Consider the function $h : x \quad \dfrac{x-2}{(x-1)^2}, x \neq 1.$

A sketch of part of the graph of *h* is given below.



The line (AB) is a vertical asymptote. The point P is a point of inflexion.

(a) Write down the **equation** of the vertical asymptote.

(1)

(b) Find $h'(x)$, writing your answer in the form

$$\frac{a-x}{(x-1)^n}$$

where a and n are constants to be determined.

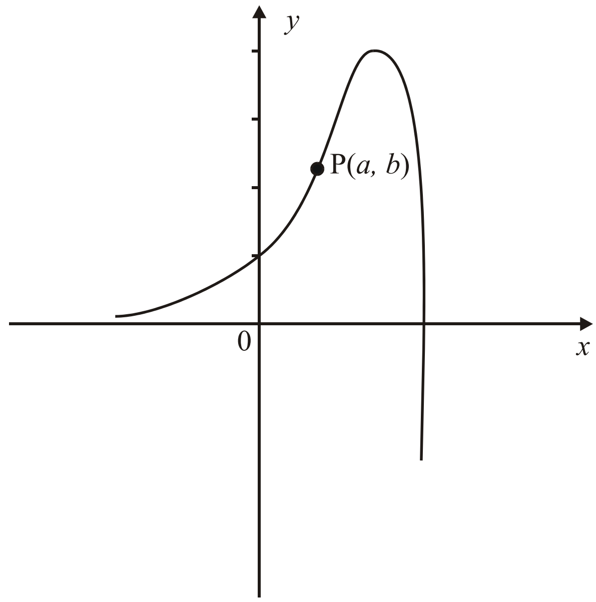
(4)

(c) Given that $h''(x) = \dfrac{2x-8}{(x-1)^4}$, calculate the coordinates of P.

(3)
(Total 8 marks)

17. The diagram shows part of the graph of the curve with equation

$$y = e^{2x} \cos x.$$



(a) Show that $\dfrac{dy}{dx} = e^{2x}(2 \cos x - \sin x)$.

(2)

(b) Find $\dfrac{d^2 y}{dx^2}$.

(4)

There is an inflexion point at P (a, b).

(c) Use the results from parts (a) and (b) to prove that:

(i) $\tan a = \dfrac{3}{4}$;

(3)

(ii) the gradient of the curve at P is e^{2a}.

(5)
(Total 14 marks)

E. Behaviour of first and second derivatives

18. The function $g(x)$ is defined for $-3 \leq x \leq 3$. The behaviour of $g'(x)$ and $g''(x)$ is given in the tables below.

x	$-3 < x < -2$	-2	$-2 < x < 1$	1	$1 < x < 3$
$g'(x)$	negative	0	positive	0	negative

x	$-3 < x < -\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2} < x < 3$
$g''(x)$	positive	0	negative

Use the information above to answer the following. In each case, justify your answer.

(a) Write down the value of x for which g has a maximum. **(2)**

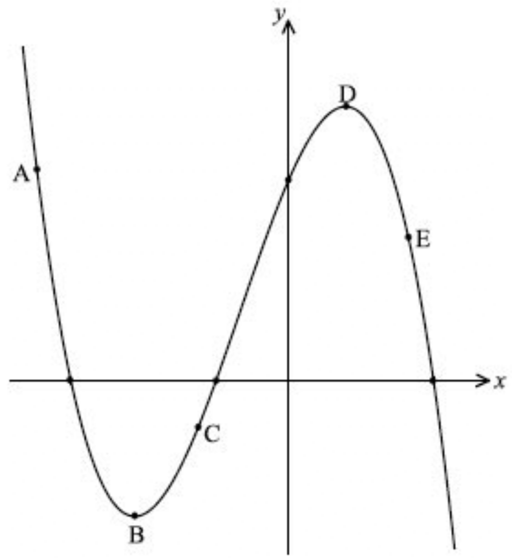
(b) On which intervals is the value of g decreasing? **(2)**

(c) Write down the value of x for which the graph of g has a point of inflexion. **(2)**

(d) Given that $g(-3) = 1$, sketch the graph of g. On the sketch, clearly indicate the position of the maximum point, the minimum point, and the point of inflexion. **(3)**

(Total 9 marks)

19. The following diagram shows part of the curve of a function f. The points A, B, C, D and E lie on the curve, where B is a minimum point and D is a maximum point.



(a) Complete the following table, noting whether $f'(x)$ is positive, negative or zero at the given points.

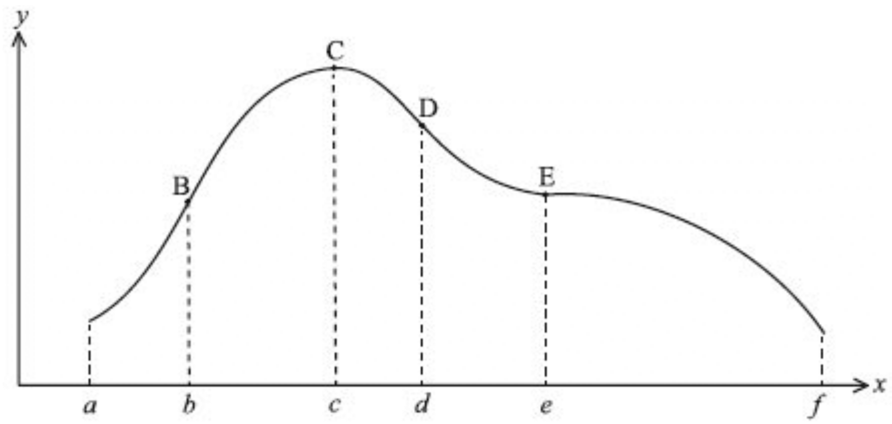
	A	B	E
$f'(x)$			

(b) Complete the following table, noting whether $f''(x)$ is positive, negative or zero at the given points.

	A	C	E
$f''(x)$			

(Total 6 marks)

20. The graph of a function g is given in the diagram below.



The gradient of the curve has its maximum value at point B and its minimum value at point D. The tangent is horizontal at points C and E.

(a) Complete the table below, by stating whether the first derivative g' is positive or negative, and whether the second derivative g'' is positive or negative.

Interval	g'	g''
$a < x < b$		
$e < x < f$		

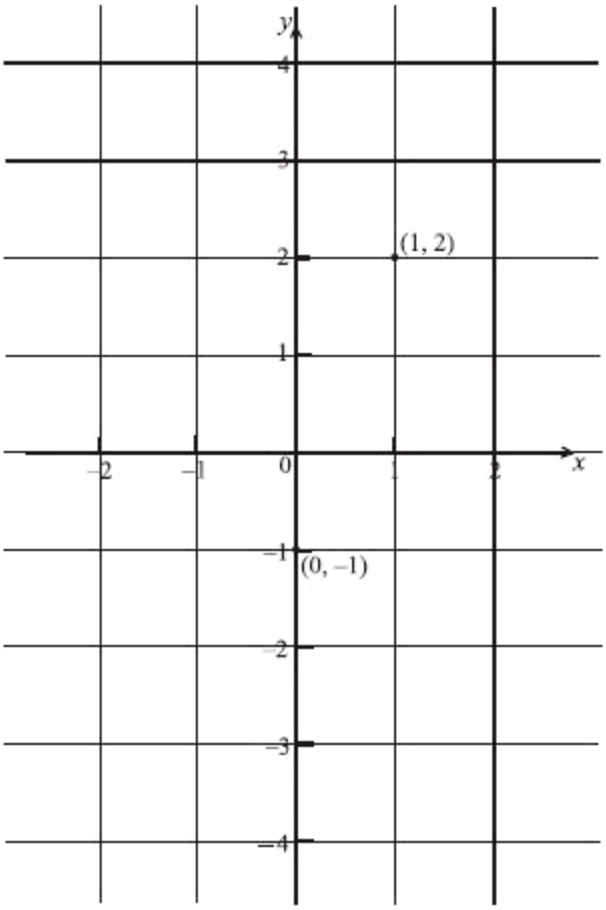
(b) Complete the table below by noting the points on the graph described by the following conditions.

Conditions	Point
$g'(x) = 0,\ g''(x) < 0$	
$g'(x) < 0,\ g''(x) = 0$	

(Total 6 marks)

21. On the axes below, sketch a curve $y = f(x)$ which satisfies the following conditions.

x	$f(x)$	$f'(x)$	$f''(x)$
$-2 \le x < 0$		negative	positive
0	–1	0	positive
$0 < x < 1$		positive	positive
1	2	positive	0
$1 < x \le 2$		positive	negative



(Total 6 marks)

F. Tangents and Normals

22. Find the equation of the normal to the curve with equation

$$y = x^3 + 1$$

at the point (1, 2).

Working:

Answer:

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(Total 4 marks)

23. A curve has equation $y = x(x - 4)^2$.

(a) For this curve find

(i) the x-intercepts;

(ii) the coordinates of the maximum point;

(iii) the coordinates of the point of inflexion.

(9)

(b) Use your answers to part (a) to sketch a graph of the curve for $0 \leq x \leq 4$, clearly indicating the features you have found in part (a).

(3)
(Total 12 marks)

24. Find the coordinates of the point on the graph of $y = x^2 - x$ at which the tangent is parallel to the line $y = 5x$.

Working:

Answer:

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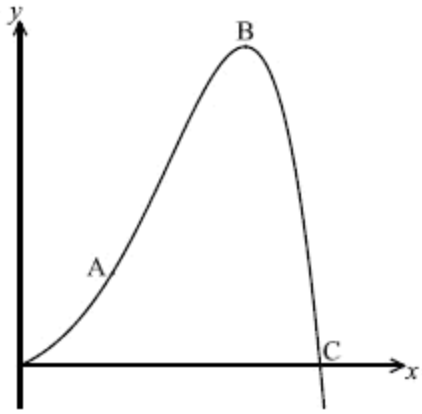
(Total 4 marks)

25. The function f is defined as $f(x) = e^x \sin x$, where x is in radians. Part of the curve of f is shown below.



There is a point of inflexion at A, and a local maximum point at B. The curve of f intersects the x-axis at the point C.

(a) Write down the x-coordinate of the point C.

(1)

(b) (i) Find $f'(x)$.

(ii) Write down the value of $f'(x)$ at the point B.

(4)

(c) Show that $f''(x) = 2e^x \cos x$.

(2)

(d) (i) Write down the value of $f''(x)$ at A, the point of inflexion.

(ii) Hence, calculate the coordinates of A.

(4)

(Total 11 marks)

26. The point P $(\frac{1}{2}, 0)$ lies on the graph of the curve of $y = \sin(2x - 1)$.

Find the gradient of the tangent to the curve at P.

Working:

Answer:

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(Total 4 marks)

27. Consider the function $f : x \quad 3x^2 - 5x + k$.

(a) Write down $f'(x)$.

The equation of the tangent to the graph of f at $x = p$ is $y = 7x - 9$. Find the value of

(b) p;

(c) k.

(Total 6 marks)

28. Let $f(x) = 3 \cos 2x + \sin^2 x$.

(a) Show that $f'(x) = -5 \sin 2x$.

(b) In the interval $\frac{\pi}{4} \le x \le \frac{3\pi}{4}$, one normal to the graph of f has equation $x = k$.

Find the value of k.

(Total 6 marks)

29. Let $f(x) = x^3 - 3x^2 - 24x + 1$.

The tangents to the curve of f at the points P and Q are parallel to the x-axis, where P is to the left of Q.

(a) Calculate the coordinates of P and of Q.

Let N_1 and N_2 be the normals to the curve at P and Q respectively.

(b) Write down the coordinates of the points where

(i) the tangent at P intersects N_2;

(ii) the tangent at Q intersects N_1.

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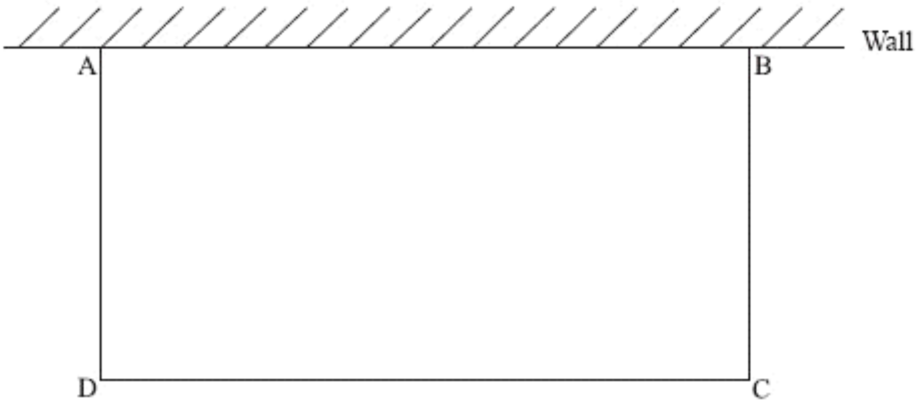
(Total 6 marks)

G. Applications of max and min

30. The following diagram shows a rectangular area ABCD enclosed on three sides by 60 m of fencing, and on the fourth by a wall AB.



Find the width of the rectangle that gives its maximum area.

Working:

Answers:

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(Total 6 marks)