Differentiation and its applications 1

1) A curve has the equation $y = xe^{2x}$.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [5]

(ii) Show that the y-coordinate of the stationary point of the curve is $-\frac{1}{2e}$. [3]

(iii) Determine the nature of this stationary point. [2]

2) A curve has the equation $y = Ae^{2x} + Be^{-x}$ where $x \geqslant 0$. At the point where $x = 0$, $y = 50$ and $\frac{dy}{dx} = -20$.

(i) Show that $A = 10$ and find the value of B. [5]

(ii) Using the values of A and B found in part **(i)**, find the coordinates of the stationary point on the curve. [4]

(iii) Determine the nature of the stationary point, giving a reason for your answer. [2]

3) Differentiate, with respect to x,

(i) $(1 - 2x)^{20}$, [2]

(ii) $x^2 \ln x$, [3]

(iii) $\frac{\tan(2x + 1)}{x}$. [3]

4) A solid circular cylinder has radius r cm and height h cm. The volume of the cylinder is 1000 cm^3.

(i) Find an expression for h in terms of r. [2]

(ii) Hence show that the total surface area, A cm^2, of the cylinder is given by

$$A = 2\pi r^2 + \frac{2000}{r}.$$ [2]

(iii) Given that r varies, find, correct to 2 decimal places, the value of r when A has a stationary value. [4]

(iv) Find this stationary value of A and determine its nature. [3]

5) The equation of a curve is $y = xe^{-\frac{x}{2}}$.

(i) Show that $\frac{dy}{dx} = \frac{1}{2}(2 - x)e^{-\frac{x}{2}}$. [3]

(ii) Find an expression for $\frac{d^2y}{dx^2}$. [2]

The curve has a stationary point at M.

(iii) Find the coordinates of M. [2]

(iv) Determine the nature of the stationary point at M. [2]