Differentiation and its applications 2

1) Differentiate with respect to x

(i) $\sqrt{1+x^3}$, [2]

(ii) $x^2 \cos 2x$. [3]

2) A curve has equation $y = \dfrac{\ln x}{x^2}$, where $x > 0$.

(i) Find the exact coordinates of the stationary point of the curve. [6]

(ii) Show that $\dfrac{d^2y}{dx^2}$ can be written in the form $\dfrac{a \ln x + b}{x^4}$, where a and b are integers. [3]

(iii) Hence, or otherwise, determine the nature of the stationary point of the curve. [2]

3) Given that a curve has equation $y = x^2 + 64\sqrt{x}$, find the coordinates of the point on the curve where $\dfrac{d^2y}{dx^2} = 0$. [7]

4) Given that $y = \dfrac{x+2}{(4x+12)^{\frac{1}{2}}}$, show that $\dfrac{dy}{dx} = \dfrac{k(x+4)}{(4x+12)^{\frac{3}{2}}}$, where k is a constant to be found. [5]

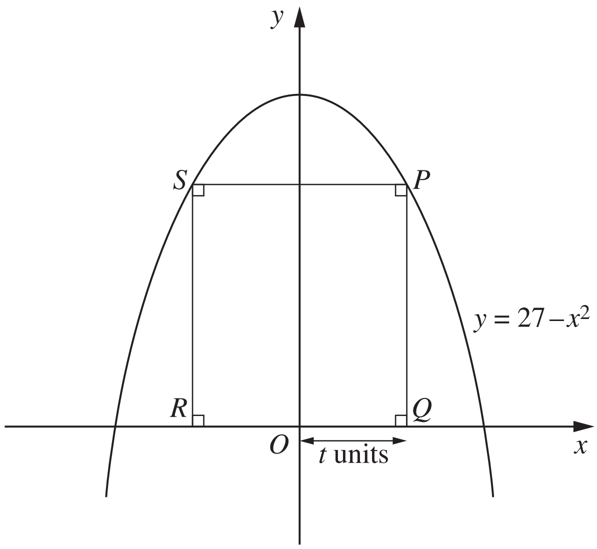
5) **(i)** Find $\dfrac{d}{dx}\left(xe^{3x} - \dfrac{e^{3x}}{3}\right)$. [3]

6) A curve has equation $y = \dfrac{2x}{x^2+9}$.

(i) Find the x-coordinate of each of the stationary points of the curve. [4]

(ii) Given that x is increasing at the rate of 2 units per second, find the rate of increase of y when $x = 1$. [3]

7)



The diagram shows part of the curve $y = 27 - x^2$. The points P and S lie on this curve. The points Q and R lie on the x-axis and $PQRS$ is a rectangle. The length of OQ is t units.

(i) Find the length of PQ in terms of t and hence show that the area, A square units, of $PQRS$ is given by

$$A = 54t - 2t^3.$$ [2]

(ii) Given that t can vary, find the value of t for which A has a stationary value. [3]

(iii) Find this stationary value of A and determine its nature. [3]