Differentiation and its applications 2 Answers

1)	**(i)** $\frac{1}{2}3x^2(1+x^3)^{-\frac{1}{2}}$	B1,B1 [2]	B1 for $\frac{1}{2}(1+x^3)^{-\frac{1}{2}}$ B1 for $\times 3x^2$
	(ii) $2x\cos 2x - 2x^2\sin 2x$	M1 A2,1,0 [3]	M1 for attempt to differentiate a product –1 each error
2)	**(i)** $\frac{dy}{dx} = \frac{x^2(\frac{1}{x}) - 2x\ln x}{x^4}$ $= \frac{1-2\ln x}{x^3}$	B3,2,1,0	–1 each error
	when $\frac{dy}{dx} = 0$, $\ln x = \frac{1}{2}$, $x = e^{\frac{1}{2}}$, $y = \frac{\frac{1}{2}}{e}$, $y = \frac{1}{2e}$	M1 A1 A1 [6]	M1 for attempt to solve $\frac{dy}{dx} = 0$
	(ii) $\frac{d^2y}{dx^2} = \frac{x^3(-\frac{2}{x}) - (1-2\ln x)3x^2}{x^6}$ $= \frac{-5+6\ln x}{x^4}$	M1 A1, A1 [3]	M1 for attempt at 2nd derivative A1 for *a*, A1for *b*
	(iii) when $x = e^{\frac{1}{2}}$, $\frac{d^2y}{dx^2}$ is –ve $(= \frac{-2}{e^2})$, max	M1 A1 [2]	M1 for a correct method A1 must be from correct working only

3)	$2x + 32x^{-\frac{1}{2}}$	B1+B1	
	$2 - 16x^{-\frac{3}{2}}$	B1+B1	
	Equate to 0 and solve	M1	
	$x = 4$	A1	
	$y = 144$	A1	[7]

4)	**(i)** $\frac{d}{dx}(4x+12)^{\frac{1}{2}} = \frac{1}{2}(4x+12)^{-\frac{1}{2}} \times 4$ or $\frac{d}{dx}(4x+12)^{-\frac{1}{2}} = \frac{-1}{2}(4x+12)^{-\frac{3}{2}} \times 4$	B1	
	Uses quotient rule or product rule	M1	
	$\frac{(4x+12)^{\frac{1}{2}} - 2(x+2)(4x+12)^{-\frac{1}{2}}}{4x+12}$ or $(4x+12)^{-\frac{1}{2}} - 2(x+2)(4x+12)^{-\frac{3}{2}}$	A1	
	Express with common denominator of $(4x+12)^n$	M1	
	$\frac{2(x+4)}{(4x+12)^{\frac{3}{2}}}$ or $k = 2$	A1	

5) **(i)** $3xe^{3x} + e^{3x} - e^{3x}$
$= 3xe^{3x}$

M1, A1, B1
[3]

M1 for attempt to differentiate a product.
A1 for correct product.
B1 for $-e^{3x}$

(ii) $\int xe^{3x}dx = \frac{1}{3}\left(xe^{3x} - \frac{e^{3x}}{3}\right)$

DM1
DM1
A1
[3]

DM1 for recognition of the 'reverse' to **(i)**
DM1 for dealing with '3'
A1 all correct (condone omission of *c*)

6) **(i)** $\frac{dy}{dx} = \frac{(x^2+9)2 - 2x(2x)}{(x^2+9)^2}$

B2,1,0

Attempt to differentiate a quotient
–1 each error

$= \frac{18 - 2x^2}{(x^2+9)^2}$, turning points,

M1

M1 for correct attempt to find the turning points.

$x = \pm 3$

A1
[4]

A1 for both

(ii) $\frac{dx}{dt} = 2$

B1

B1 for use of $\frac{dx}{dt} = 2$

$\frac{dy}{dt} = 2 \times \left(\frac{16}{100}\right)$

M1

M1 for use of rates of change

$= 0.32$ or $\frac{8}{25}$

A1
[3]

7)

(i) $27 - t^2$ B1
$2t \times (27 - t^2)$ B1

(ii) $\frac{dA}{dt} = 54 - 6t^2$ B1

Solve $\frac{dA}{dt} = 0$ M1

$t = 3$ A1

(iii) Substitute for t in expression for A M1
$A = 108$ only A1
completely correct method and maximum B1

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