Differentiation and its applications Answers

1)

(i) $y = xe^{2x}$ d/dx(e^{2x}) = $2e^{2x}$	B1	Anywhere – even if product not used
→ dy/dx = $e^{2x} + 2x\,e^{2x}$	M1A1	Use of correct formula for "uv". co
→ $d^2y/dx^2 = 2e^{2x} + 2e^{2x} + 4xe^{2x}$	M1A1 [5]	Use of product formula again. co.
(ii) dy/dx = 0 when $1+2x = 0$ → $x = -½$	M1 A1	Sets his dy/dx to 0 and tries to solve.
→ $y = -½e^{-1} = -\dfrac{1}{2e}$.	A1 [3]	co – ag – beware fortuitous results.
(iii) If $x = -½$ → +ve result → Minimum	M1 A1	Looks at sign. Correct deduction from correct x.
or gradient goes −,0,+)		(or by any other valid method)
or y value to left or right of $(-½) > -\frac{1}{2e}$)	[2]	

2)

(i) $50 = A + B$	B1	
$\dfrac{dy}{dx} = 2Ae^{2x} - Be^{-x}$	M1	M1 for attempt to differentiate
	A1	A1 all correct
$-20 = 2A - B$	DM1	DM1 for attempt to solve equations.
leads to $A = 10$ and $B = 40$	A1 [5]	
(ii) $\dfrac{dy}{dx} = 20e^{2x} - 40e^{-x}$, $20e^{2x} = 40e^{-x}$	M1	M1 for equating to zero and attempt at solution
$e^{3x} = 2$	M1	M1 for dealing with exponentials
$x = \dfrac{1}{3}\ln 2$ or 0.231	M1	M1 for attempt to obtain x
$y = 47.6$	A1 [4]	A1 for both
(iii) $\dfrac{d^2y}{dx^2} = 40e^{2x} + 40e^{-x}$	M1	M1 for attempt at second derivative or other valid method
Always +ve, so min	A1 [2]	A1 for a correct conclusion

3)

9 (i) $20 \times -2(1-2x)^{19}$	B1,B1 [2]	B1 for 20 and $(1-2x)^{19}$ B1 for -2
(ii) $x^2\dfrac{1}{x} + 2x\ln x$	M1	M1 for attempt to differentiate a product.
	B1	B1 for $\dfrac{1}{x}$
	A1 [3]	
(iii) $\dfrac{x\left(2\sec^2(2x+1)\right) - \tan(2x+1)}{x^2}$	M1	M1 for attempt to differentiate a quotient.
	B1	B1 for differentiation of $\tan(2x+1)$
	A1 [3]	

4)

(i) $\pi r^2 h = 1000$, leading to $h = \dfrac{1000}{\pi r^2}$	M1 A1 [2]	M1 for attempt to use volume
(ii) $A = 2\pi rh + 2\pi r^2$ leading to given answer $A = 2\pi r^2 + \dfrac{2000}{r}$	M1 A1 [2]	M1for attempt to use surface area GIVEN ANSWER
(iii) $\dfrac{dA}{dr} = 4\pi r - \dfrac{2000}{r^2}$ when $\dfrac{dA}{dr} = 0,\ \ 4\pi r = \dfrac{2000}{r^2}$ leading to $r = 5.42$	M1 A1 DM1 A1 [4]	M1 for attempt to differentiate and set to 0 DM1 for solution
(iv) $\dfrac{d^2A}{dr^2} = 4\pi + \dfrac{4000}{r^3}$ + ve when $r = 5.42$ so min value A_{min}=554	M1 A1 A1 [3]	M1 for second derivative method or gradient method' A1 for minimum, can be given if r incorrect but + ve

5)

(i) $d(e^{-1/2x})/dx = -\tfrac{1}{2}e^{-1/2x}$	B1
$d(xe^{-1/2x})/dx = e^{-1/2x} + x(\ldots) = \tfrac{1}{2}(2-x)e^{-1/2x}$	M1 A1
(ii) $d^2y/dx^2 = -\tfrac{1}{2}e^{-1/2x} + (-\tfrac{1}{2})(e^{-1/2x} - \tfrac{1}{2}xe^{-1/2x})$ $[= -\tfrac{1}{4}(4-x)e^{-1/2x}]$	M1 A1
(iii) $dy/dx = 0$ when $2-x=0 \Rightarrow x = 2,\ y = 2e^{-1}$ $[\approx 0.736]$	M1 A1
(iv) When $x=2$, $d^2y/dx^2 < 0$ $[= -\tfrac{1}{2}e^{-1} \approx -0.184] \Rightarrow$ maximum	M1 A1

6)

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