SL

Differentiation 2

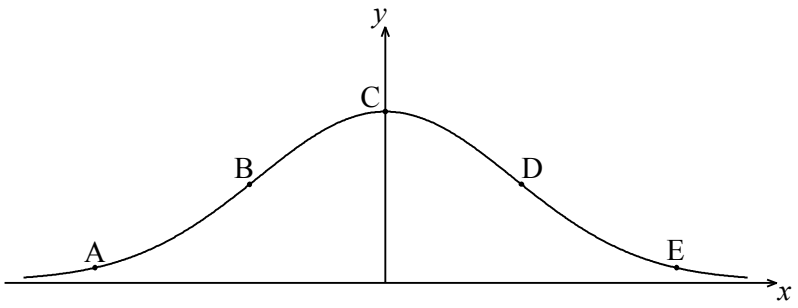
1) Let $g(x) = \frac{\ln x}{x^2}$, for $x > 0$.

(a) Use the quotient rule to show that $g'(x) = \frac{1 - 2\ln x}{x^3}$. *[4 marks]*

(b) The graph of g has a maximum point at A. Find the x-coordinate of A. *[3 marks]*

2) Let $h(x) = \frac{6x}{\cos x}$. Find $h'(0)$. *[Maximum mark: 6]*

3)calc The following diagram shows the graph of $f(x) = e^{-x^2}$.



The points A, B, C, D and E lie on the graph of f. Two of these are points of inflexion.

(a) Identify the **two** points of inflexion. *[2 marks]*

(b) (i) Find $f'(x)$.

(ii) Show that $f''(x) = (4x^2 - 2)e^{-x^2}$. *[5 marks]*

(c) Find the x-coordinate of each point of inflexion. *[4 marks]*

(d) Use the second derivative to show that one of these points is a point of inflexion. *[4 marks]*

4)calc Let $f(x) = \frac{20x}{e^{0.3x}}$, for $0 \le x \le 20$.

(a) Sketch the graph of f. *[3 marks]*

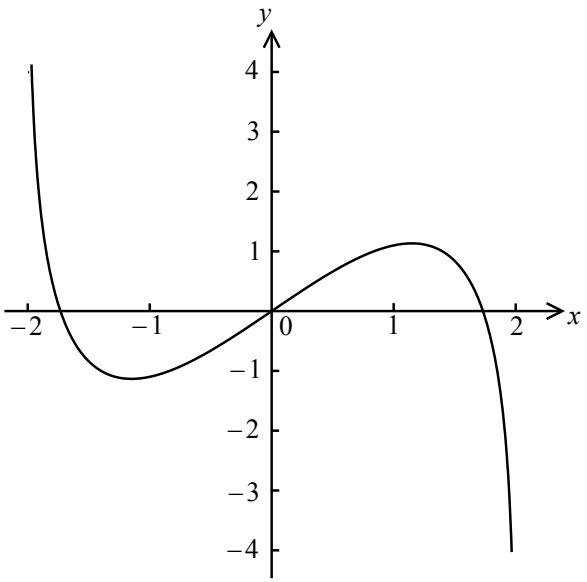
(b) (i) Write down the x-coordinate of the maximum point on the graph of f.

(ii) Write down the interval where f is increasing. *[3 marks]*

(c) Show that $f'(x) = \frac{20 - 6x}{e^{0.3x}}$. *[5 marks]*

(d) Find the interval where the rate of change of f is increasing. *[4 marks]*

5)calc Consider $f(x) = x\ln(4 - x^2)$, for $-2 < x < 2$. The graph of f is given below.



(a) Let P and Q be points on the curve of f where the tangent to the graph of f is parallel to the x-axis.

(i) Find the x-coordinate of P and of Q.

(ii) Consider $f(x) = k$. Write down all values of k for which there are exactly two solutions. *[5 marks]*

Let $g(x) = x^3\ln(4 - x^2)$, for $-2 < x < 2$.

(b) Show that $g'(x) = \frac{-2x^4}{4 - x^2} + 3x^2\ln(4 - x^2)$. *[4 marks]*

(c) Sketch the graph of g'. *[2 marks]*

(d) Consider $g'(x) = w$. Write down all values of w for which there are exactly two solutions. *[3 marks]*

Let $f'(x) = -24x^3 + 9x^2 + 3x + 1$.

6)calc (a) There are two points of inflexion on the graph of f. Write down the x-coordinates of these points. *[3 marks]*

(b) Let $g(x) = f''(x)$. Explain why the graph of g has no points of inflexion. *[2 marks]*