Differentiation 2 ANSWERS

1)

5. (a) $\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}$, $\frac{\mathrm{d}}{\mathrm{d}x}x^2 = 2x$ (seen anywhere) ***A1A1***

attempt to substitute into the quotient rule (do **not** accept product rule) ***M1***

e.g. $\frac{x^2\left(\frac{1}{x}\right)-2x\ln x}{x^4}$

correct manipulation that clearly leads to result ***A1***

e.g. $\frac{x-2x\ln x}{x^4}$, $\frac{x(1-2\ln x)}{x^4}$, $\frac{x}{x^4}-\frac{2x\ln x}{x^4}$

$g'(x)=\frac{1-2\ln x}{x^3}$ ***AG*** ***N0***

[4 marks]

(b) evidence of setting the derivative equal to zero ***(M1)***

e.g. $g'(x)=0$, $1-2\ln x=0$

$\ln x = \frac{1}{2}$ ***A1***

$x=\mathrm{e}^{\frac{1}{2}}$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

2)

4. **METHOD 1 (quotient)**

derivative of numerator is 6 ***(A1)***
derivative of denominator is $-\sin x$ ***(A1)***

attempt to substitute into quotient rule ***(M1)***

correct substitution ***A1***

e.g. $\frac{(\cos x)(6)-(6x)(-\sin x)}{(\cos x)^2}$

substituting $x=0$ ***(A1)***

e.g. $\frac{(\cos 0)(6)-(6\times 0)(-\sin 0)}{(\cos 0)^2}$

$h'(0)=6$ ***A1*** ***N2***

[6 marks]

METHOD 2 (product)

$h(x)=6x\times(\cos x)^{-1}$

derivative of $6x$ is 6 ***(A1)***
derivative of $(\cos x)^{-1}$ is $\left(-(\cos x)^{-2}(-\sin x)\right)$ ***(A1)***

attempt to substitute into product rule ***(M1)***
correct substitution ***A1***

e.g. $(6x)\left(-(\cos x)^{-2}(-\sin x)\right)+(6)(\cos x)^{-1}$

substituting $x=0$ ***(A1)***

e.g. $(6\times 0)\left(-(\cos 0)^{-2}(-\sin 0)\right)+(6)(\cos 0)^{-1}$

$h'(0)=6$ ***A1*** ***N2***

[6 marks]

3)

9. (a) B, D *A1A1* *N2*

[2 marks]

(b) (i) $f'(x) = -2x\mathrm{e}^{-x^2}$ *A1A1* *N2*

Note: Award *A1* for e^{-x^2} and *A1* for $-2x$.

(ii) finding the derivative of $-2x$, *i.e.* -2 *(A1)*

evidence of choosing the product rule *(M1)*

e.g. $-2\mathrm{e}^{-x^2} - 2x \times -2x\mathrm{e}^{-x^2}$

$-2\mathrm{e}^{-x^2} + 4x^2\mathrm{e}^{-x^2}$ *A1*

$f''(x) = (4x^2 - 2)\mathrm{e}^{-x^2}$ *AG* *N0*

[5 marks]

(c) valid reasoning *R1*

e.g. $f''(x) = 0$

attempting to solve the equation *(M1)*

e.g. $(4x^2 - 2) = 0$, sketch of $f''(x)$

$p = 0.707 \left(= \frac{1}{\sqrt{2}}\right)$, $q = -0.707 \left(= -\frac{1}{\sqrt{2}}\right)$ *A1A1* *N3*

[4 marks]

(d) evidence of using second derivative to test values on either side of POI *M1*

e.g. finding values, reference to graph of f'', sign table

correct working *A1A1*

e.g. finding any two correct values either side of POI,
checking sign of f'' on either side of POI

reference to sign change of $f''(x)$ *R1* *N0*

[4 marks]

Total [15 marks]

4)

10. (a)

A1A1A1 ***N3***

Note: Award ***A1*** for approximately correct shape with inflexion/ change of curvature, ***A1*** for maximum skewed to the left, ***A1*** for asymptotic behaviour to the right.

[3 marks]

(b) (i) $x = 3.33$ ***A1*** ***N1***

(ii) correct interval, with right end point $3\frac{1}{3}$ ***A1A1*** ***N2***

e.g. $0 < x \le 3.33,\ 0 \le x < 3\frac{1}{3}$

Note: Accept any inequalities in the right direction.

[3 marks]

(c) valid approach ***(M1)***
e.g. quotient rule, product rule

2 correct derivatives (must be seen in product or quotient rule) ***(A1)(A1)***
e.g. $20,\ 0.3\mathrm{e}^{0.3x}$ or $-0.3\mathrm{e}^{-0.3x}$

correct substitution into product or quotient rule ***A1***

e.g. $\frac{20\mathrm{e}^{0.3x} - 20x(0.3)\mathrm{e}^{0.3x}}{(\mathrm{e}^{0.3x})^2},\ 20\mathrm{e}^{-0.3x} + 20x(-0.3)\mathrm{e}^{-0.3x}$

correct working ***A1***

e.g. $\frac{20\mathrm{e}^{0.3x} - 6x\mathrm{e}^{0.3x}}{\mathrm{e}^{0.6x}},\ \frac{\mathrm{e}^{0.3x}\left(20 - 20x(0.3)\right)}{(\mathrm{e}^{0.3x})^2},\ \mathrm{e}^{-0.3x}\left(20 + 20x(-0.3)\right)$

$f'(x) = \frac{20 - 6x}{\mathrm{e}^{0.3x}}$ ***AG*** ***N0***

[5 marks]

continued …

(d) consideration of f' or f'' ***(M1)***

valid reasoning ***R1***
e.g. sketch of f', f'' is positive, $f'' = 0$, reference to minimum of f'

correct value $6.6666666\ldots \left(6\frac{2}{3}\right)$ ***(A1)***

correct interval, with **both** end points ***A1*** ***N3***

e.g $6.67 < x \le 20,\ 6\frac{2}{3} \le x < 20$

[4 marks]

Total [15 marks]

5)

QUESTION 10

(a) (i) $-1.15,\ 1.15$ ***A1A1*** ***N2***

(ii) recognizing that it occurs at P and Q ***(M1)***
e.g. $x = -1.15,\ x = 1.15$

$k = -1.13,\ k = 1.13$ ***A1A1*** ***N3***

[5 marks]

(b) evidence of choosing the product rule ***(M1)***
e.g. $uv' + vu'$

derivative of x^3 is $3x^2$ ***(A1)***

derivative of $\ln(4 - x^2)$ is $\frac{-2x}{4 - x^2}$ ***(A1)***

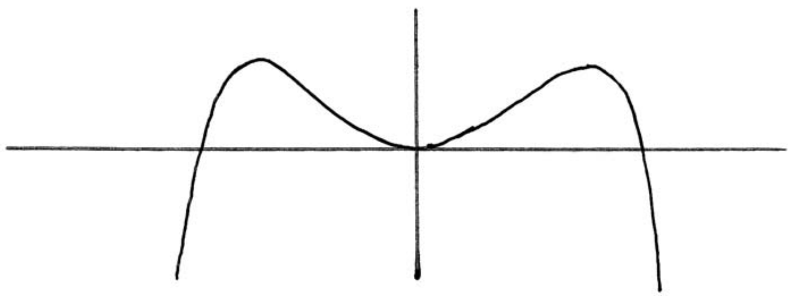
correct substitution ***A1***

e.g. $x^3 \times \frac{-2x}{4 - x^2} + \ln(4 - x^2) \times 3x^2$

$g'(x) = \frac{-2x^4}{4 - x^2} + 3x^2 \ln(4 - x^2)$ ***AG*** ***N0***

[4 marks]

(c)



A1A1 ***N2***

[2 marks]

(d) $w = 2.69, \quad w < 0$ ***A1A2*** ***N2***

[3 marks]

Total [14 marks]

6)

(a) valid approach *R1*

e.g. $f''(x) = 0$, the max and min of f' gives the points of inflexion on f

$-0.114,\ 0.364$ $\left(\text{accept } (-0.114,\ 0.811) \text{ and } (0.364,\ 2.13)\right)$ *A1A1* *N1N1*

(b) **METHOD 1**

graph of g is a quadratic function *R1* *N1*

a quadratic function does not have any points of inflexion *R1* *N1*

METHOD 2

graph of g is concave down over entire domain *R1* *N1*

therefore no change in concavity *R1* *N1*

METHOD 3

$g''(x) = -144$ *R1* *N1*

therefore no points of inflexion as $g''(x) \neq 0$ *R1* *N1*

[5 marks]