Differentiation

80 min

77 marks

1. Consider the function $g(x) = x^4 + 3x^3 + 2x^2 + x + 4$.

Find

(a) $g'(x)$

(3)

(b) $g'(1)$

(2)

(Total 5 marks)

2. Consider the function $f(x) = 2x^3 - 5x^2 + 3x + 1$.

(a) Find $f'(x)$.

(3)

(b) Write down the value of $f'(2)$.

(1)

(c) Find the equation of the tangent to the curve of $y = f(x)$ at the point (2, 3).

(2)

(Total 6 marks)

3. Let $f(x) = 2x^2 + x - 6$
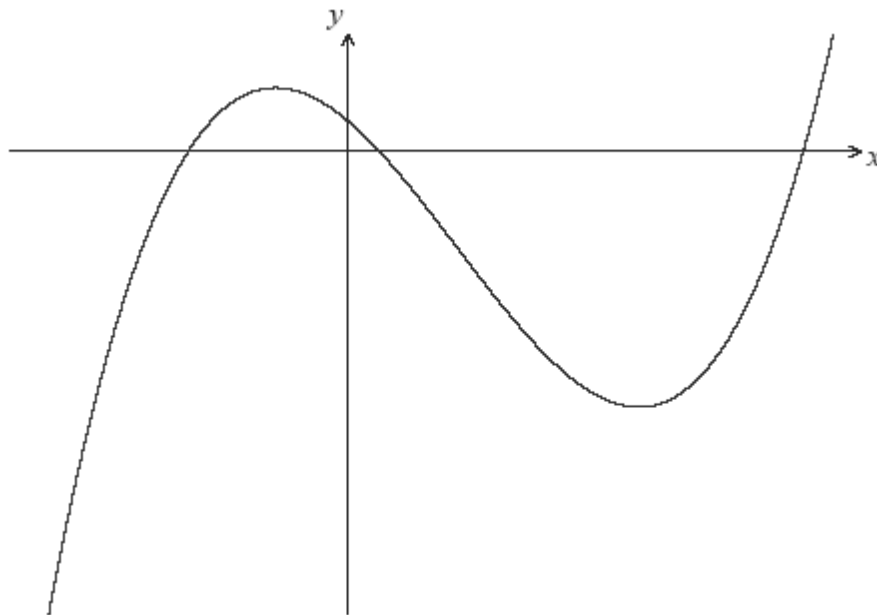
   (a) Find $f'(x)$.

   (3)

   (b) Find the value of $f'(-3)$.

   (1)

   (c) Find the value of x for which $f'(x) = 0$.

   (Total 6 marks)

4. The diagram shows a sketch of the function $f(x) = 4x^3 - 9x^2 - 12x + 3$.

   diagram not to scale

   (a) Write down the values of x where the graph of $f(x)$ intersects the x-axis.

   (3)

   (b) Write down $f'(x)$.

   (3)

(c)   Find the value of the local maximum of $y = f(x)$.

(4)

Let P be the point where the graph of $f(x)$ intersects the y-axis.

(d)   Write down the coordinates of P.

(1)

(e)   Find the gradient of the curve at P.

(2)

The line, L, is the tangent to the graph of $f(x)$ at P.

(f)   Find the equation of L in the form $y = mx + c$.

(2)

There is a second point, Q, on the curve at which the tangent to $f(x)$ is parallel to L.

(g)   Write down the gradient of the tangent at Q.

(1)

(h)   Calculate the x-coordinate of Q.

(3)
(Total 19 marks)

5.   (a)   Differentiate the following function with respect to x:

$$f(x) = 2x - 9 - 25x^{-1}$$

(b)   Calculate the x-coordinates of the points on the curve where the gradient of the tangent to the curve is equal to 6.

(Total 6 marks)

6. A function $g(x) = x^3 + 6x^2 + 12x + 18$

(a) Find $g'(x)$.

(3)

(b) Solve $g'(x) = 0$.

(2)

(c) (i) Calculate the values of $g'(x)$ when

   (a) $x = -3$;

   (b) $x = 0$.

(ii) Hence state whether the function is increasing or decreasing at

   (a) $x = -3$;

   (b) $x = 0$.

(4)
(Total 9 marks)

7. The straight line, L, has equation $2y - 27x - 9 = 0$.

(a) Find the gradient of L.

(2)

Sarah wishes to draw the tangent to $f(x) = x^4$ parallel to L.

(b) Write down $f'(x)$.

(1)

(c) (i) Find the x-coordinate of the point at which the tangent must be drawn.

(ii) Write down the value of $f(x)$ at this point.

(3)
(Total 6 marks)

8.   *It is **not** necessary to use graph paper for this question.*

(a)   Sketch the curve of the function $f(x) = x^3 - 2x^2 + x - 3$ for values of x from -2 to 4, giving the intercepts with both axes.

(3)

(b)   On the same diagram, sketch the line $y = 7 - 2x$ and find the coordinates of the point of intersection of the line with the curve.

(3)

(c)   Find the value of the gradient of the curve where $x = 1.7$.

(2)

(Total 8 marks)

9.   (a)   Differentiate the function $y = x^2 + 3x - 2$.

(b)   At a certain point (x, y) on this curve the gradient is 5. Find the co-ordinates of this point.

(Total 6 marks)

10.   Consider the function $f(x) = \dfrac{1}{2}x^3 - 2x^2 + 3$.

(a)   Find $f'(x)$.

(2)

(b)   Find $f''(x)$.

(2)

(c)   Find the equation of the tangent to the curve of f at the point $(1, 1.5)$.

(2)

(Total 6 marks)