cos/sine rule ans studies

0 min
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1. *Unit penalty (UP) is applicable in question part (a)* ***only***.

(a) $AC^2 = 625^2 + 986^2 - 2 \times 625 \times 986 \times \cos 102°$ (M1)(A1)

(= 1619072.159)

AC = 1272.43

UP = 1270m (A1) (C3)

(b) $\frac{986}{\sin A} = \frac{1270}{\sin 102°}$ (M1)(A1)(ft)

A = 49.4° (A1)(ft)

OR

$\frac{986}{\sin A} = \frac{1272.43}{\sin 102°}$ (M1)(A1)(ft)

A = 49.3° (A1)(ft)

OR

$\cos A = \left(\frac{625^2 + 1270^2 - 986^2}{2 \times 625 \times 1270}\right)$ (M1)(A1)(ft)

A = 49.5° (A1)(ft) (C3)

[6]

2. (a) $C\hat{A}B = 180 - 2 \times 23°$ (M1)

$= 134°$ (A1) (C2)

(b) $\dfrac{AB}{\sin 23°} = \dfrac{15}{\sin 134°}$ (M1)

Note: *Follow through with candidate's answer from (a)*

$AB = \dfrac{15 \sin 23°}{\sin 134°}$

$AB = 8.147702831...$

$= 8.15$ (3 s.f.) (A1) (C2)

[4]

3. (a) $BC = \sqrt{48^2 + 57^2 - 2(48)(57)\cos 117°}$ (or equivalent) (M1)

≈ 89.7 m (3 s.f.) (A1)

(b) Area of $\Delta ABC = \dfrac{1}{2} ab \sin C = \dfrac{1}{2}(48)(57)\sin 117°$ (M1)

$= 1220\ \text{m}^2$ (3 s.f.) (A1)

[4]

4. (a) AC = 19 – 11 = 8 (M1)

$6^2 = 5^2 + 8^2 - 2(5)(8)\cos \text{B}\hat{\text{A}}\text{C}$ (M1)

$\Rightarrow \text{B}\hat{\text{A}}\text{C} = 48.5°$ (3 s.f.) (A1) 3

(b) Area $= \left(\dfrac{1}{2}\right)(5)(8)\sin \text{B}\hat{\text{A}}\text{C}$ (M1)

$= 15.0\ \text{cm}^2$ (3 s.f.) (allow **ft** from part (a)) (A1) 2

[5]

5. ***Unit penalty applies in part (b).***

(a) $\sin \text{A}\hat{\text{B}}\text{D} = \dfrac{4}{9}$ (M1)

100 + their $(\text{A}\hat{\text{B}}\text{D})$ (M1)

126° (A1) (C3)

Notes: *Accept an equivalent trigonometrical equation involving angle ABD for the first (M1).*
Radians used gives 100°. Award at most (M1)(M1)(A0) if working shown.
BD = 8 m leading to 127°. Award at most (M1)(M1)(A0) (premature rounding).

(b) $AC^2 = 10^2 + 9^2 - 2 \times 10 \times 9 \times \cos(126.38...)$ (M1)(A1)

Notes: *Award (M1) for substituted cosine formula.*
Award (A1) for correct substitution using their answer to part (a).

UP AC = 17.0 m (A1)(ft) (C3)

Notes: *Accept 16.9 m for using 126.*
Follow through from their answer to part (a).
Radians used gives 5.08. Award at most (M1)(A1)(A0)(ft) if working shown.

[6]

6. (a) (i) 8.5 (cm) (A1)

(ii) 120° (A1)

(iii) 30° (A1) (C3)

(b) $\frac{BC}{\sin 120°} = \frac{8.5}{\sin 30°}$ (M1)(A1)(ft)

Note: *Award (M1) for correct substituted formula, (A1) for correct substitutions.*

BC = 14.7 $\left(\frac{17\sqrt{3}}{2}\right)$ (A1)(ft) (C3)

[6]

7. ***Unit penalty applies in parts (b) and (c)***

(a) 60° (A1) (C1)

UP (b) $\dfrac{15\times\sqrt{15^2-7.5^2}}{2}$ = 97.4 cm^2 (97.5 cm^2) (A1)(M1)(A1)

Notes: *Award (A1) for correct height, (M1) for substitution in the area formula, (A1) for correct answer. Accept 97.5 cm^2 from taking the height to be 13 cm.*

OR

$\dfrac{1}{2}\times 15^2\times\sin 60° = 97.4$ cm^2 (M1)(A1)(A1)(ft) (C3)

Notes: *Award (M1) for substituted formula of the area of a triangle, (A1) for correct substitution, (A1)(ft) for answer. Follow through from their answer to part (a). If radians used award at most (M1)(A1)(A0)***.**

UP (c) 97.4 × 120 **=** 11700 cm^3 (M1)(A1)(ft) (C2)

Notes: *Award (M1) for multiplying their part (b) by 120.*

[6]

8. (a) $\dfrac{\sin A\hat{B}C}{13.4}=\dfrac{\sin 30°}{6.7}$ (M1)(A1)

Note: *Award (M1) for correct substituted formula, (A1) for correct substitution.*

$A\hat{B}C = 90°$ (A1)

$A\hat{C}B = 60°$ (A1)(ft) (C4)

Note: *Radians give no solution, award maximum (M1)(A1)(A0).*

(b) $\dfrac{29-30}{30}\times 100$ (M1)

Note: *Award (M1) for correct substitution into correct formula.*

% error = –3.33 % (A1) (C2)

Note: *Percentage symbol not required. Accept positive answer.*

[6]

9. ***Note: Unit penalty (UP) applies in parts (b)(c) and (e)***

(a) $\dfrac{\sin \text{BCA}}{35} = \dfrac{\sin 105°}{80}$ (M1)(A1)

Note: *Award (M1) for correct substituted formula, (A1) for correct substitutions.*

$\text{B}\hat{\text{C}}\text{A} = 25.0°$ (A1)(G2)

(b) Length BD = 40 m (A1)

Angle ABC = 180° – 105° – 25° = 50° (A1)(ft)

Note: *(ft) from their answer to (a).*

$\text{AD}^2 = 35^2 + 40^2 - (2 \times 35 \times 40 \times \cos 50°)$ (M1)(A1)(ft)

Note: *Award (M1) for correct substituted formula, (A1)(ft) for correct substitutions.*

UP AD = 32.0m (A1)(ft)(G3)

Notes: *If 80 is used for BD award at most (A0)(A1)(ft)(M1)(A1)(ft)(A1)(ft) for an answer of 63.4 m.*
If the angle ABC is incorrectly calculated ***in this part*** *award at most (A1)(A0)(M1)(A1)(ft)(A1)(ft).*
If angle BCA is used award at most (A1)(A0)(M1)(A0)(A0).

(c) length of fence = 35 + 40 + 32 (M1)

UP = 107m (A1)(ft)(G2)

Note: *(M1) for adding 35 + 40 + their (b).*

(d) cost per metre = $\dfrac{802.50}{107}$ (M1)

Note: *Award (M1) for dividing 802.50 by their (c).*

cost per metre = 7.50 USD (7.5 USD) (USD not required) (A1)(ft)(G2)

(e) Area of ABD = $\dfrac{1}{2} \times 35 \times 40 \times \sin 50°$ (M1)

= 536.2311102 (A1)(ft)

UP = 536m^2 (A1)(ft)(G2)

Note: *Award (M1) for correct substituted formula, (A1)(ft) for correct substitution, (ft) from their value of BD and their angle ABC in (b).*

(f) Volume = 0.03×536 (A1)(M1)

$= 16.08$

$= 16.1$ (A1)(ft)(G2)

Note: *Award (A1) for 0.03, (M1) for correct formula.*
(ft) from their (e).
If 3 is used award at most (A0)(M1)(A0).

[18]

10. (a) $AC^2 = 3.9^2 + 4.8^2 - 2 \times 3.9 \times 4.8 \times \cos 82°$ (M1)(A1)

$AC^2 = 33.04$

$AC = 5.75$ (A1) 3

(b) $\dfrac{3.9}{\sin C} = \dfrac{\sqrt{33.04}}{\sin 82°}$ (M1)(A1)

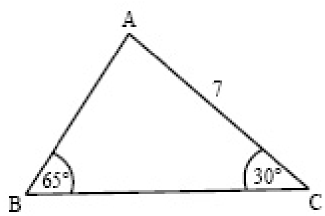
$\sin C = 0.671889$

$C = 42.2°$ (A1) 3

[6]

11. *Unit penalty **(UP)** may apply in this question.*

(a)



(A1)

Note: *(A1) for fully labelled sketch.* (C1)

(b) $\dfrac{AB}{\sin 30} = \dfrac{7}{\sin 65}$ (M1)

UP $AB = 3.86$ cm (A1)(ft)

Note: *(M1) for use of sine rule with correct values substituted.* (C2)

(c) Angle $\hat{BAC} = 85°$ (A1)

Area = $\frac{1}{2} \times 7 \times 3.86 \times \sin 85°$ (M1)

UP $= 13.5 \text{ cm}^2$ (A1)(ft) (C3)

[6]

12. *Unit penalty **(UP)** is applicable where indicated.*

(a) $\frac{1}{2} 20^2 \sin B = 100$ (M1)(A1)

$B = 30°$ (AG)

***Note:** (M1) for correct substituted formula and (A1) for correct substitution.*
B=30° must be seen or previous (A1) mark is lost.

(b) $\overline{AC}^2 = 2 \times 20^2 - 2 \times 20^2 \times \cos 30°$ (M1)(A1)

UP $\overline{AC} = 10.4 \text{ cm}$ (A1)(G2)

***Note:** (M1) for using cosine rule, (A1) for correct substitution.*
Last (A1) is for the correct answer. Accept use of sine rule or any correct method e.g. AC = 2 × 20 sin 15°.

[5]

13. ***Note on use of radians:***

In (a) the answer will be –874. Award (A0) at the last step for either + or – 874.
In (b) follow through with either sign from (a) can receive (M1) (A1) ft, but in both cases the final answer of ±947000 receives (A0) for unrealistic sign and/or for unrealistic magnitude.

(a) Third angle of triangle =180 – (75 + 40) (M1)
= 65° (A1)

***Notes:** Award (A2) for 65 seen.*
For use of 40° or 75° in an otherwise correct sine rule award (M1)(A0)(A0)

Length of fence: $\frac{x}{\sin 65°} = \frac{410}{\sin 75°}$ (sine rule) (M1)(A1)

x = 385 m (3 s.f.) (A1)
or (G2) 5

(b) Area = $\frac{1}{2}ab \sin c$

area = $\frac{1}{2} \times 385 \times 245 \sin 24°$ (M1)(A1)

= 19 200 (m^2) (3 s.f.) (A1)

or (G2) 3

[8]

14. (a) Area = $\frac{1}{2} \times 14 \times 8 \sin 110°$ (M1)

= 52.62278676 m^2

= 52.6 m^2 (3s.f) (A1)

(b) $\frac{\sin C}{8} = \frac{\sin 110°}{18}$ (or equivalent) (M1)

$\sin C = \frac{8 \times \sin 110°}{18}$

C = 24.68575369

C = 24.7° (3s.f.) (A1)

Note: *Accept all answers obtained from all appropriate methods, given to the correct degree of accuracy.*

[4]

15. (a) $PR^2 = 7.8^2 + 11.1^2 - 2 \times 7.8 \times 11.1 \times \cos 102°$ (M1)

= 60.84 + 123.21 – (–36.00)

= 220.05

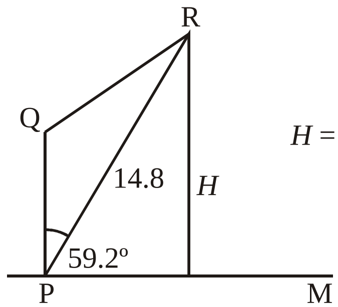
PR = 14.8 m (or $\sqrt{220.05}$) (A1) 2

(b) $\frac{11.1}{\sin \hat{R}} = \frac{14.8}{\sin 102°}$ (Follow through with candidate's answer to part (a))

$\Rightarrow \sin \hat{R} = \frac{11.1 \sin 102°}{14.8} = 0.7336$ (M1)

$\Rightarrow \hat{R} = 47.2°$ (or 47.0° from $\sqrt{220.05}$) (A1) 2

(c)



$H = h + 6.5$

Angle $Q\hat{P}R = 180° – (102° + 47.2°)$

$= 30.8°$ (or $31.0°$) (M1)

$\Rightarrow R\hat{P}M = 90° – 30.8° = 59.2°$ (or $59.0°$)

$\sin 59.2° = \dfrac{H}{14.8}$ (M1)

$\Rightarrow H = 14.8 \sin 59.2° = 12.7$ m

OR

$\cos 30.8° = \dfrac{H}{14.8}$ (M1)

$\Rightarrow H = 14.8 \cos 30.8° = 12.7$ m

Therefore, $h = 12.7 – 6.5$
$= 6.2$ m (A1) 3

[7]

16. (a) $BD^2 = 15^2 + 20^2 – 2 \times 15 \times 20 \times \cos 110°$ (M1)(A1)

Note: *Award (M1) for using the cosine rule, award (A1) for correct substitution.*

$BD^2 = 830.212$
$BD = 28.8$ (A1)

OR

$BD = 28.8$ (G3) 3

(b) $\dfrac{28.81}{\sin C} = \dfrac{22}{\sin 30°}$ (M1)(A1)

$C = 40.9°$ (G1)

OR

$C = 40.9°$ (G3) 3

(c) $BD = 30$ (A1) 1

(d) $\frac{30}{\sin C} = \frac{22}{\sin 30°}$ (M1)

$C = 43.0°$ (A1)

OR

$C = 43.0°$ (G2) 2

(e) Percentage error = $\frac{43.0 - 40.9}{40.9} \times 100$ (M1)(A1)

= 5.13% (A1) 3

[12]