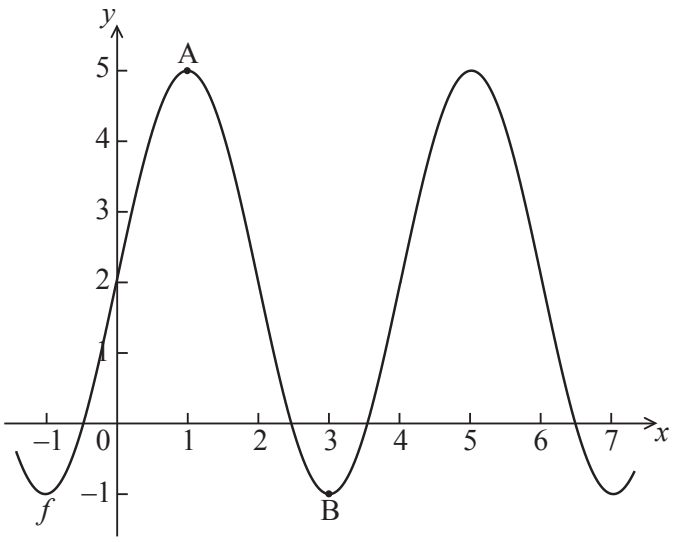
Circular functions and Trig test

NON CALCULATOR SECTION

1) The diagram below shows part of the graph of a function f.



The graph has a maximum at $\mathrm{A}(1, 5)$ and a minimum at $\mathrm{B}(3, -1)$.

The function f can be written in the form $f(x) = p\sin(qx) + r$. Find the value of

(a) p; *[2 marks]*

(b) q; *[2 marks]*

(c) r. *[2 marks]*

2) Let $\sin 100^\circ = m$. Find expressions for each of the following in terms of m

(a) $\cos 100^\circ$; *[3 marks]*

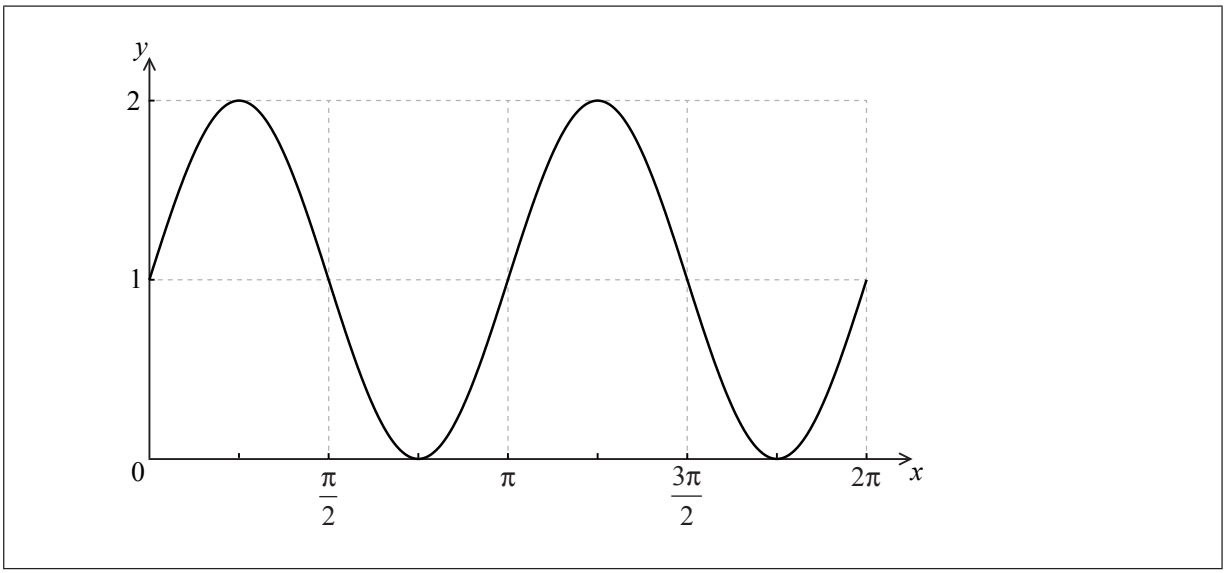
(b) $\tan 100^\circ$; *[1 mark]*

(c) $\sin 200^\circ$. *[2 marks]*

3) Let $f(x) = (\sin x + \cos x)^2$.

(a) Show that $f(x)$ can be expressed as $1 + \sin 2x$. *[2 marks]*

The graph of f is shown below for $0 \le x \le 2\pi$.



(b) Let $g(x) = 1 + \cos x$. On the same set of axes, sketch the graph of g for $0 \le x \le 2\pi$. *[2 marks]*

The graph of g can be obtained from the graph of f under a horizontal stretch of scale factor p followed by a translation by the vector $\begin{pmatrix} k \\ 0 \end{pmatrix}$.

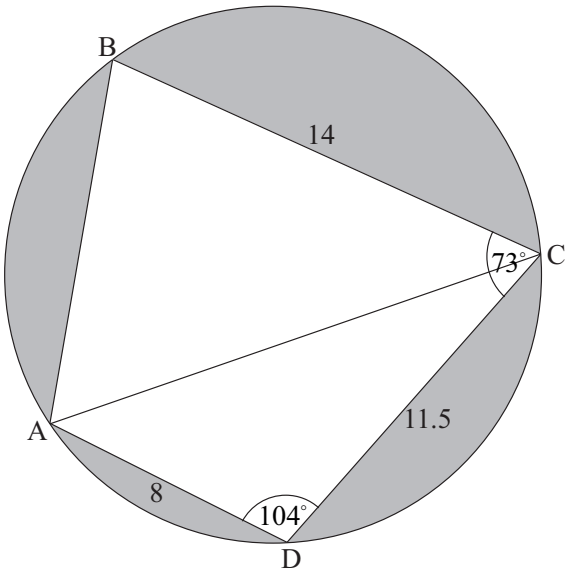
(c) Write down the value of p and a possible value of k. *[2 marks]*

4) Solve the equation $2\cos^2 x = \sin 2x$ for $0 \le x \le \pi$, giving your answers in terms of π.

CALCULATOR SECTION

5) *[Maximum mark: 14]*

The diagram shows a circle of radius 8 metres. The points ABCD lie on the circumference of the circle.



BC = 14 m, CD = 11.5 m, AD = 8 m, $A\hat{D}C = 104^\circ$, and $B\hat{C}D = 73^\circ$

(a) Find AC. *[3 marks]*

(b) (i) Find $A\hat{C}D$.

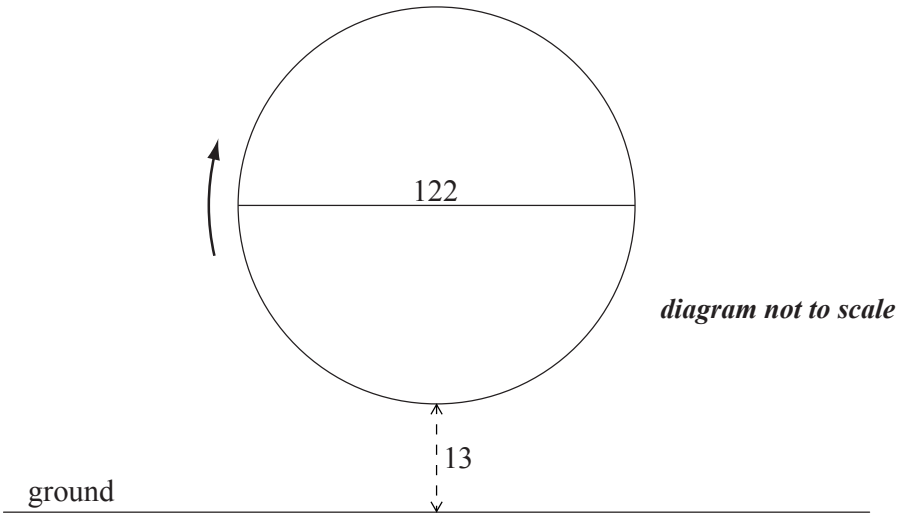
(ii) Hence, find $A\hat{C}B$. *[5 marks]*

(c) Find the area of triangle ADC. *[2 marks]*

(d) Hence or otherwise, find the total area of the shaded regions. *[4 marks]*

6)

A Ferris wheel with diameter 122 metres rotates clockwise at a constant speed. The wheel completes 2.4 rotations every hour. The bottom of the wheel is 13 metres above the ground.



A seat starts at the bottom of the wheel.

(a) Find the maximum height above the ground of the seat. *[2 marks]*

After t minutes, the height h metres above the ground of the seat is given by

$$h = 74 + a\cos bt\ .$$

(b) (i) Show that the period of h is 25 minutes.

(ii) Write down the **exact** value of b. *[2 marks]*

(c) Find the value of a. *[3 marks]*

(d) Sketch the graph of h, for $0 \le t \le 50$. *[4 marks]*

(e) In one rotation of the wheel, find the probability that a randomly selected seat is at least 105 metres above the ground. *[5 marks]*