Circular functions and Trig test

**NON CALCULATOR SECTION**

1)

(a) valid approach to find $p$ ***(M1)***

*eg* amplitude $= \frac{\max - \min}{2}$, $p = 6$

$p = 3$ ***A1*** ***N2***

***[2 marks]***

(b) valid approach to find $q$ ***(M1)***

*eg* period $= 4$, $q = \frac{2\pi}{\text{period}}$

$q = \frac{\pi}{2}$ ***A1*** ***N2***

***[2 marks]***

(c) valid approach to find $r$ ***(M1)***

*eg* axis $= \frac{\max + \min}{2}$, sketch of horizontal axis, $f(0)$

$r = 2$ ***A1*** ***N2***

***[2 marks]***

***Total [6 marks]***

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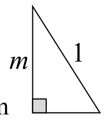
2)

**METHOD 1**

(a) valid approach involving Pythagoras *(M1)*




*e.g.* $\sin^2 x + \cos^2 x = 1$, labelled diagram

correct working (may be on diagram) *(A1)*

*e.g.* $m^2 + (\cos 100)^2 = 1$, $\sqrt{1-m^2}$

$\cos 100 = -\sqrt{1-m^2}$ ***A1*** ***N2***

***[3 marks]***

(b) $\tan 100 = -\frac{m}{\sqrt{1-m^2}}$ $\left(\text{accept } \frac{m}{-\sqrt{1-m^2}}\right)$ ***A1*** ***N1***

***[1 mark]***

(c) valid approach involving double angle formula *(M1)*

*e.g.* $\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 200 = -2m\sqrt{1-m^2}$ $\left(\text{accept } 2m\left(-\sqrt{1-m^2}\right)\right)$ ***A1*** ***N2***

**Note:** If candidates find $\cos 100 = \sqrt{1-m^2}$, award full ***FT*** in parts (b) and (c), even though the values may not have appropriate signs for the angles.

***[2 marks]***

***Total [6 marks]***

**METHOD 2**

(a) valid approach involving tan identity *(M1)*

*e.g.* $\tan = \frac{\sin}{\cos}$

correct working *(A1)*

*e.g.* $\cos 100 = \frac{\sin 100}{\tan 100}$

$\cos 100 = \frac{m}{\tan 100}$ ***A1*** ***N2***

***[3 marks]***

*continued ...*

(b) $\tan 100 = \frac{m}{\cos 100}$ ***A1*** ***N1***

***[1 mark]***

(c) valid approach involving double angle formula *(M1)*

*e.g.* $\sin 2\theta = 2\sin\theta\cos\theta$, $2m \times \frac{m}{\tan 100}$

$\sin 200 = \frac{2m^2}{\tan 100}$ $(= 2m\cos 100)$ ***A1*** ***N2***

***[2 marks]***

***Total [6 marks]***

3)

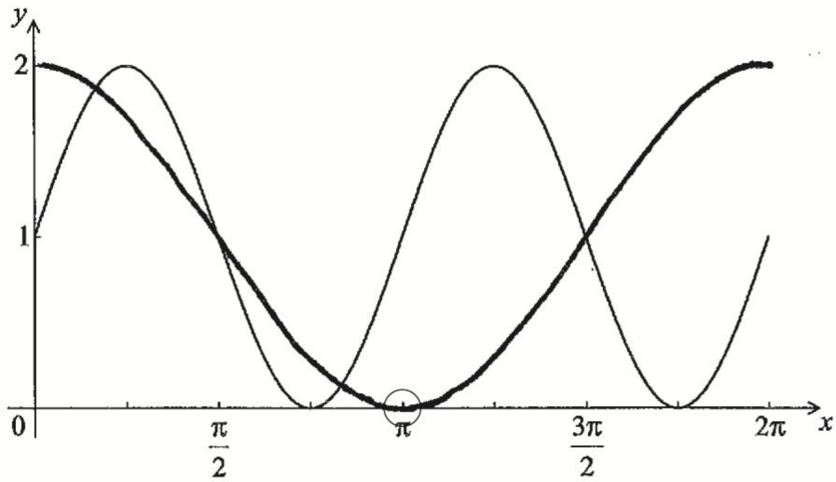
(a) attempt to expand *(M1)*
*e.g.* $(\sin x+\cos x)(\sin x+\cos x)$; at least 3 terms

correct expansion ***A1***
*e.g.* $\sin^2 x+2\sin x\cos x+\cos^2 x$

$f(x)=1+\sin 2x$ ***AG*** ***N0***

***[2 marks]***

(b)




***A1A1*** ***N2***

| | |
|---|---|
| **Note:** | Award ***A1*** for correct sinusoidal shape with period 2π and range [0, 2], ***A1*** for minimum in circle. |

***[2 marks]***

(c) $p=2,\ k=-\frac{\pi}{2}$ ***A1A1*** ***N2***

***[2 marks]***

***Total [6 marks]***

4)

**METHOD 1**

$2\cos^2 x = 2\sin x \cos x$ ***(M1)***

$2\cos^2 x - 2\sin x \cos x = 0$

$2\cos x(\cos x - \sin x) = 0$ ***(M1)***

$\cos x = 0, (\cos x - \sin x) = 0$ ***(A1)(A1)***

$x = \frac{\pi}{2}, x = \frac{\pi}{4}$ ***(A1)(A1)*** ***(C6)***

**METHOD 2**

Graphical solutions

**EITHER**

for both graphs $y = 2\cos^2 x$, $y = \sin 2x$, ***(M2)***

**OR**

for the graph of $y = 2\cos^2 x - \sin 2x$. ***(M2)***

**THEN**

Points representing the solutions clearly indicated ***(A1)***

1.57, 0.785 ***(A1)***

$x = \frac{\pi}{2}, x = \frac{\pi}{4}$ ***(A1)(A1)*** ***(C6)***

**Notes:** If no working shown, award ***(C4)*** for one correct answer.
Award ***(C2)(C2)*** for each correct decimal answer 1.57, 0.785.
Award ***(C2)(C2)*** for each correct degree answer $90^\circ, 45^\circ$.
Penalize a total of ***[1 mark]*** for any additional answers.

Circular functions and Trig test

**CALCULATOR SECTION**

5)

(a) evidence of choosing cosine rule *(M1)*

*eg* $c^2 = a^2 + b^2 - 2ab\cos C$, $\text{CD}^2 + \text{AD}^2 - 2 \times \text{CD} \times \text{AD}\cos D$

correct substitution ***A1***

*eg* $11.5^2 + 8^2 - 2 \times 11.5 \times 8\cos 104$, $196.25 - 184\cos 104$

$\text{AC} = 15.5\,(\text{m})$ ***A1*** ***N2***

***[3 marks]***

(b) (i) **METHOD 1**

evidence of choosing sine rule *(M1)*

*eg* $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$, $\dfrac{\sin \text{A}\hat{\text{C}}\text{D}}{\text{AD}} = \dfrac{\sin D}{\text{AC}}$

correct substitution ***A1***

*eg* $\dfrac{\sin \text{A}\hat{\text{C}}\text{D}}{8} = \dfrac{\sin 104}{15.516\ldots}$

$\text{A}\hat{\text{C}}\text{D} = 30.0^\circ$ ***A1*** ***N2***

**METHOD 2**

evidence of choosing cosine rule *(M1)*

*eg* $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution ***A1***

*eg* $8^2 = 11.5^2 + 15.516\ldots^2 - 2(11.5)(15.516\ldots)\cos C$

$\text{A}\hat{\text{C}}\text{D} = 30.0^\circ$ ***A1*** ***N2***

(ii) subtracting **their** $\text{A}\hat{\text{C}}\text{D}$ from 73 *(M1)*

*eg* $73 - \text{A}\hat{\text{C}}\text{D}$, $70 - 30.017...$

$\text{A}\hat{\text{C}}\text{B} = 43.0^\circ$ ***A1*** ***N2***

***[5 marks]***

(c) correct substitution *(A1)*

*eg* area $\Delta\,\text{ADC} = \dfrac{1}{2}(8)(11.5)\sin 104$

area $= 44.6\ (\text{m}^2)$ ***A1*** ***N2***

***[2 marks]***

(d) attempt to subtract *(M1)*

*eg* circle − ABCD, $\pi r^2 - \Delta\,\text{ADC} - \Delta\,\text{ACB}$

area $\Delta\,\text{ACB} = \dfrac{1}{2}(15.516\ldots)(14)\sin 42.98\ (= 74.0517\ldots)$ *(A1)*

correct working ***A1***

*eg* $\pi(8)^2 - 44.6336\ldots - \dfrac{1}{2}(15.516\ldots)(14)\sin 42.98$, $64\pi - 44.6 - 74.1$

shaded area is 82.4 ($\text{m}^2$) ***A1*** ***N3***

***[4 marks]***

***Total [14 marks]***

6)

(a) valid approach *(M1)*
*eg* $13+\text{diameter},\ 13+122$

maximum height $=135$ (m) ***A1*** ***N2***
***[2 marks]***

(b) (i) $\text{period}=\dfrac{60}{2.4}$ ***A1***

$\text{period}=25$ (minutes) ***AG*** ***N0***

(ii) $b=\dfrac{2\pi}{25}\quad(=0.08\pi)$ ***A1*** ***N1***

***[2 marks]***

(c) **METHOD 1**

valid approach *(M1)*
*eg* $\max-74\ ,\ |a|=\dfrac{135-13}{2}\ ,\ 74-13$

$|a|=61$ (accept $a=61$) *(A1)*

$a=-61$ ***A1*** ***N2***
***[3 marks]***

**METHOD 2**

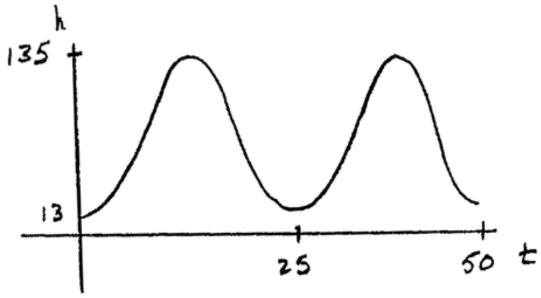
attempt to substitute valid point into equation for $h$ *(M1)*
*eg* $135=74+a\cos\left(\dfrac{2\pi\times 12.5}{25}\right)$

correct equation *(A1)*
*eg* $135=74+a\cos(\pi)\ ,\ 13=74+a$

$a=-61$ ***A1*** ***N2***
***[3 marks]***

*continued …*

(d)




*A1A1A1A1* *N4*

| |
|---|
| **Note:** Award ***A1*** for approximately correct domain, ***A1*** for approximately correct range, ***A1*** for approximately correct sinusoidal shape with 2 cycles. **Only** if this last ***A1*** awarded, award ***A1*** for max/min in approximately correct positions. |

***[4 marks]***

(e) setting up inequality (accept equation) ***(M1)***

*eg* $h > 105$, $105 = 74 + a\cos bt$, sketch of graph with line $y = 105$

any **two** correct values for $t$ (seen anywhere) ***A1A1***

*eg* $t = 8.371\ldots$, $t = 16.628\ldots$, $t = 33.371\ldots$, $t = 41.628\ldots$,

valid approach ***M1***

*eg* $\frac{16.628 - 8.371}{25}, \frac{t_1 - t_2}{25}, \frac{2 \times 8.257}{50}, \frac{2(12.5 - 8.371)}{25}$

$p = 0.330$ ***A1*** ***N2***

***[5 marks]***

***Total [16 marks]***