IB Questionbank Mathematical Studies 3rd edition

Chi Sq MS

0 min 0 marks

1. (a)
$$\frac{40}{120} \left(\frac{1}{3}, 0.333, 33.3\% \right)$$
 (A1)(A1)(G2)

Note: Award (A1) for numerator, (A1) for denominator.

(b)
$$\frac{34}{120} \left(\frac{17}{60}, 0.283, 28.3\% \right)$$
 (A1)(A1)(G2)

Note: Award (A1) for numerator, (A1) for denominator.

(c)
$$\frac{8}{28} \left(\frac{2}{7}, 0.286, 28.6\%\right)$$
 (A1)(A1)(G2)

Note: Award (A1) for numerator, (A1) for denominator.

(d) customer satisfaction is **independent** of café (A1) *Note:* Accept "customer satisfaction is **not associated with** the café".

(f) 0.754 (G2) *Note:* Award (G1)(G1)(AP) for 0.75 or for correct answer incorrectly rounded to 3 s.f. or more, (G0) for 0.7.

ince $\chi^2_{calc} < \chi^2_{crit}$ (5.991) accept (or Do not reject) H ₀ Note: Follow through from their value in (e).	(R1)(A1)(ft)
DR	
Accept (or Do not reject) H_0 as <i>p</i> -value (0.686) > 0.05 <i>Notes:</i> Do not award (A1)(R0). Award the (R1) for comparison of appropriate	(R1)(A1)(ft)
Awara the (K1) for comparison of appropriate	values.
H_0 : Choice of language is independent of gender.	(A1)

2.

(c)
$$\frac{50 \times 69}{150} = 23$$
 (M1)(A1)(G2)

Notes: Award (M1) for correct substituted formula, (A1) for 23.

- (d) $\chi^2 = 4.77$ (G2) **Notes:** If answer is incorrect, award (M1) for correct substitution in the correct formula (all terms).
- (e) Accept H₀ since $\chi^2_{calc} < \chi^2_{crit}$ (5.99) or *p*-value (0.0923) > 0.05 (R1)(A1)(ft) Notes: Do not award (R0)(A1). Follow through from their (d) and (b).

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3.	(a)	H ₀ : The (average) number of meals per day a student has and
		gender are independent (A1)
		<i>Note:</i> For "independent" accept "not associated" but do not accept "not related" or "not correlated".
	(b)	2 (A1)

- (c) 5.99 (accept 5.991) (A1)(ft) Note: Follow through from their part (b).

(d)	$\frac{28 \times 45}{100} = 12.6 = 13 \text{ or } \frac{28}{100} \times \frac{25}{100} \times 100 = 12.6 = 13 \tag{M1}$	A1)(AG)
	<i>Notes:</i> Award (M1) for correct formula and (A1) for co substitution. Unrounded answer must be seen for the (A awarded.	
(e)	0.0321 Note: For 0.032 award (G1)(G1)(AP). For 0.03 with no working award (G0).	(G2)
(f)	0.0321 < 5.99 or 0.984 > 0.05	(R1)

accept H₀

Note: If reason is incorrect both marks are lost, do not award (R0)(A1).

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(A1)(ft)

4. (a) H_0 : The height of the rice plants is independent of the use of a fertilizer. (A1)

Notes: For independent accept "not associated", can accept "the use of a fertilizer has no effect on the height of the plants". Do not accept "not correlated".

H₁: The height of the rice plants is not independent (dependent) of the use of fertilizer. (A1)(ft)

Note: If H_0 and H_1 are reversed award (A0)(A1)(ft).

(b) $\frac{180 \times 195}{360}$ or $\frac{180}{360} \times \frac{195}{360} \times 360$ (A1)(A1)(M1) = 97.5 (AG) *Notes: Award (A1) for numerator, (A1) for denominator (M1) for division. If final 97.5 is not seen award at most (A1)(A0)(M1).*

(c)
$$\chi^2_{calc} = 14.01 \ (14.0, 14)$$
 (G2)

OR

If worked out by hand award (*M1*) for correct substituted formula with correct values, (*A1*) for correct answer. (M1)(A1)

- (d) 2 (A1)
- (e) $\chi^2_{crit} = 9.210 \ (9.21)$ (A1)(ft)

Note: (*ft*) from candidates answer to (*d*).

(f)	$\chi^2_{calc} > \chi^2_{crit}$	(R1)
	The manufacturer's claim is justified. (or equivalent statement)	(A1)

Note: Do not accept (R0)(A1).

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- (iii) 5.991 (5.99) (A1)(ft)
 Note: Follow through from (c)(ii) for their degrees of freedom.
 Note: Accept any accuracy beyond 3 s.f.
- (iv) Accept the null hypothesis since 1.367 < 5.991 (A1)(ft)(R1)
 Note: Allow "Do not reject". Follow through from their null hypothesis and their critical value.
 Full credit for use of p-values from GDC [p = 0.505]
 Do not award (A1)(R0). Award (R1) for valid comparison.

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6. (a) (i)
$$\frac{280}{400} (0.7,70\% \text{ or equivalent})$$
 (A1)(A1)(G2)
Note: (A1) for correct numerator, (A1) for correct denominator.

(ii)
$$\frac{57}{210} \left(\frac{19}{70}, 0.271, 27.1\%\right)$$
 (A1)(A1)(G2)

Note: (A1) for correct numerator, (A1) for correct denominator.

(b)
$$\frac{180}{400} \times \frac{179}{399}$$
 (A1)(M1)

Note: (A1) for correct values seen, (M1) for multiplying their two values (A1) for correct answer.

$$=\frac{537}{2660}(=0.202)\tag{A1}(G3)$$

(c) H_0 : "the preference of brand of cereal is independent of the city".

OR (A1)

 H_0 : "there is no association between the brand of cereal and city".

$$(d) \quad df = 2 \tag{A1}$$

(e)
$$\frac{210 \times 120}{400}$$
 (M1)(A1)

Note: (*M1*) for substituting in correct formula, (*A1*) for correct values.

(g) $39.3 > \chi^2_{\text{crit}} \text{ or } p\text{-value} < 0.05$ (R1)(ft)

Do not accept H_0 . (A1)(ft) *Notes:* Allow "Reject H_0 " or equivalent.

(ft) from their χ^2 statistic.

Award (R1)(ft) for comparing the appropriate values. (A1)(ft) can be awarded only if the conclusion is valid according to the comparison given. If no reason given or if reason is wrong both marks are lost. Note that (R1)(A0)(ft)can be awarded but (R0)(A1)(ft) cannot.

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7.	(a)	(i)	H_0 = wearing of a seat belt and the time a driver has held	
			a licence are independent.	(A1)
			Note: For independent accept "not associated" but do not	
			accept "not related" or "not correlated"	

(iii)
$$\frac{98 \times 45}{200} = 22.05 = 22$$
 (correct to the nearest whole number)(M1)(A1)(AG)

Note: (*M*1) for correct formula and (*A*1) for correct substitution. Unrounded answer must be seen for the (*A*1) to be awarded.

- (iv) $\chi^2 = 8.12$ (G2) **Note:** For unrounded answer award (G1)(G0)(AP) If formula used award (M1) for correct substituted formula with correct substitution (6 terms) (A1) for correct answer.
- (v) "Does not accept H_0 " (A1)(ft)

$$\chi^2_{crit} < 8.12 \text{ or } p\text{-value} < 0.05$$
 (R1)(ft)

Note: Allow "Reject H_0 " or equivalent. Follow through from their χ^2 statistic. Award (R1)(ft) for comparing the appropriate values. The (A1)(ft) can be awarded only if the conclusion is valid according to the comparison given. If no reason given or if reason is wrong the two marks are lost.

(b) (i)
$$\frac{98}{200}$$
 (= 0.49, 49%) (A1)(A1)(G2)

Note: (A1) for numerator, (A1) for denominator.

(ii)
$$\frac{15}{45}$$
 (= 0.333, 33.3%) (A1)(A1)(G2)

Note: (A1) for numerator, (A1) for denominator.

(c) (i)
$$\frac{98}{200} \times \frac{97}{199} = 0.239 \ (23.9 \ \%)$$
 (A1)(M1)(A1) (G3)

Note: (A1) for correct probabilities seen, (M1) for multiplying two probabilities, (A1) for correct answer.

(ii)
$$1 - \frac{102}{200} \times \frac{101}{199} = 0.741 \ (74.1 \ \%)$$
 (M1)(M1)

(A1)(ft)(G2)

Note: (*M*1) for showing the product, (*M*1) for using the probability of the complement, (*A*1) for correct answer. Follow through for consistent use of with replacement.

OR

$$\frac{98}{200} \times \frac{97}{199} + \frac{98}{200} \times \frac{102}{199} + \frac{102}{200} \times \frac{98}{199} = 0.741 \ (74.1 \ \%) \tag{M1}$$

(A1)(ft)(G2)

Note: (*M*1) for adding three products of fractions (or equivalent), (*M*1) for using the correct fractions, (*A*1) for correct answer Follow through for consistent use of with replacement.

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8. (a)

	Drama	Comedy	Film	News
Males	58	119	157	52
Females	86	98	120	61

(M1)(M1)(A1) 3

(b) H_0 : favourite TV programme is independent of gender or no association between favourite TV programme and gender

H₁: favourite TV programme is dependent on gender (*must have both*) (A1) 1

(c)
$$\frac{365 \times 217}{751}$$
 (M1)

$$= 105$$
 (A1)(ft)(G2) 2

(d) 12.6 (accept 12.558) (G3) 3

(e)	(i)	3	(A1)	
	(ii)	7.815 (accept 7.82)((ft) from their (i))	(A1)(ft)	
	(iii)	reject H_0 or equivalent statement (eg accept H_1)	(A1)(ft)	3

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(a)	(i)	H_0 : level of stress is independent of travel time	(A1)
		H_1 : level of stress is not independent of travel time	(A1)(ft)
		(or reasonable equivalents)	()()
	(ii)	12.1 5.24 14.6	
		20.1 8.68 24.2	
		11.8 5.08 14.2	(M1)(A1)(G2)
		<i>Note:</i> (M1) for attempting to calculate expected val	ues by
		hand eg $\frac{44 \times 32}{116} = 12.1$ etc.	
		12 5 15	
		20 9 24	
		12 5 14	
		Nearest integers (A1)(G3)	
	(iii)	df = $(r-1)(c-1) = (3-1)(3-1) = 4$	(M1)(AG)
	(iv)	$\chi^2 = 9.83(1)$	(G2)
		OR $\chi^2 = 9.277$ if calculated from integer values	OR (M1)(A1)
	(v)	For $\chi^2 = 9.83$ Do not accept H ₀ :	(A1)(ft)
		(Level of stress is not independent of travel time or reasonal because $\chi^2_{calc} > \chi^2_{crit}$ or <i>p</i> -value < 0.05	ble equivalent) (R1)(ft)
		OR	
		For $\chi^2 = 9.278$ Accept H ₀ :	(A1)(ft)
		because $\chi^2_{calc} > \chi^2_{crit}$ or <i>p</i> -value > 0.05	(R1)(ft)
		<i>Note:</i> a correct reason must be given for the (A1) to awarded.	be
(b)	(i)	r = 0.667	(A1)
	(ii)	Stress rating increases as travel time increases (or reasonable equivalent eg y increases as x increases).	(R1)
		<i>Note:</i> Do not accept "positive correlation"	
	(iii)	y = 0.181x + 2.22	
		for 0.181 <i>x</i> and for 2.22	(A1) (A1)
		$101 \ 2.22$	(AI)

9.

Note: For y = 2.22x + 0.181, award (A0)(A1)(ft)

(iv)	Putting $x = 45$	(M1)
	$0.181 \times 45 + 2.22$	
	= 10.365 (10.4)	(A1)(ft)(G2)

Notes: Allow 10 or 11 only if the method is shown and is correct. Allow follow through only if method shown.

(v)	not reliable	(A1)
	Because result is outside the data range or because the	
	correlation coefficient not high or the sample is small or	
	responses are subjective.	(R1)
	<i>Note:</i> Award (<i>R</i> 1) for any of the above. A correct reason must be given to award the (A1).	

) 1
) 1
, 1
) 1
) 2
)

p-value of 0.106 > 0.05

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2

(R1)

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11.	(a)	H_0 : number of entries is independent of gender.	(A1)	1

(b) 4 (A1) 1

(d)
$$a = 85, b = 29$$
 (A1)(A1) 2

(e)
$$\frac{(30-32)^2}{32} + \dots$$
 (M1)(A1)

- $= 6.10 \text{ (using given values)} \tag{A1}$
- OR

5.80 (from calculator)

(f) Do not reject the null hypothesis as the χ^2 value is less than the critical value. So, gender and stroke are independent. (A1)(R1) 2 (Also allow "accept").

12.	(a)	(i)	a = 29.45, b = -5.55, c = 30.8025, d = 1.046	(A4)	
		(ii)	$\chi^2_{\text{calc}} = 1.012 + 0.946 + 1.118 + 1.046 = 4.12$	(A1)	
		(iii)	degrees of freedom = $(2-1)(2-1) = 1$	(A1)	
		(iv)	$\chi^2_{\rm crit} = 3.84$	(A1)	7

(b) Do not accept H₀. The weight of a puppy is related to the weight of the parent. (A1)(R1) 2 $\chi^2_{calc} > \chi^2_{crit}$

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[10]

3

(G3)

13.	(a)	Male = 49:	$es = \frac{9000 \times 11000}{20000}$ 50	(M1)(A1) (AG)	2
	(b)	(i)	That gender and grade obtained are independent. (There is no connection between gender and grade obtained.)	(A1)	
		(ii)	(3-1)(2-1) = 2	(A1)	
		(iii)	$\chi^2 = 5.991$	(A1)	
		(iv)	Calculated $\chi^2 = 39.957$ Therefore, reject the Null hypothesis. Gender and grade obtai are dependent (or there is a connection between gender and gr		4

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14.	(a)	(i)	Alternative hypothesis: Choice of candidates depends on voter location.(A1)			
		(ii)	a =188 b = -22	(A1) (A1)	3	

(b) (i)
$$\chi^2_{\text{calc}} = \frac{484}{317} + \frac{484}{204} + \frac{484}{291} + \frac{484}{188}$$
 (M1)
= 8.14 (accept 8.13) (A1)

$$= 8.14$$
 (accept 8.13)
Note: Award (G2) for 7.97.

(ii)
$$v = 1$$
 (A1)

(iii)
$$\chi^2(0.95, 1)$$
 (M1)
= 3.84 (A1) 5

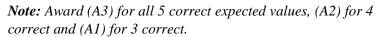
(c)(i)Where voters live does affect how they vote.(A1)(ii)
$$\chi^2_{calc} > \chi^2$$
 or $8.14 > 3.84$ so we reject the null hypothesis.(R1)

[10]

2

15. (a)

olour	%	Expected	
led	35	21	
Drange	25	15	(A3)
Green	20	12	
Purple	15	9	
Blue	5	3	



(b)	Blue It has less than 5 sweets	(A1) (R1)	2
		()	
(c)	$H_0 = Colour \text{ of sweets is a random sample}$	(A1)	
	d.o.f. = $(4 - 1)(2 - 1) = 3$	(A1)	
	critical value = 7.815	(A1)	
	$\chi^2 = \frac{(21-18)^2}{21} + \frac{(15-17)^2}{15} + \frac{(12-10)^2}{12} + \frac{(12-15)^2}{12}$	(M1)	
	= 1.78	(A1)	
	1.78 < 7.815	(A1)	
	Therefore, accept H ₀	(A1)	7

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16.	(a)	(i)	Expected number of male managers	
			$=\frac{160}{500}\times\frac{300}{500}\times500=\frac{160\times300}{500}$	(M1)(A1)
			= 96	(AG)

(ii)
$$b = 160 - 96 = 64$$
 (A1)
 $c = 300 - 96 - 60 = 144$ (A1)
 $d = 240 - 144 = 96$ (A1)
(A1) 5

(b)	(i)	H ₀ : Position is independent of gender		
	(ii)	H ₁ : Position is dependent on gender	(A1)	2

(c) (i)
$$\chi^2_{\text{calc}} = \frac{(96-95)^2}{96} + \frac{(64-65)^2}{64} + \frac{(114-130)^2}{144} + \frac{(96-110)^2}{96} + \frac{(60-75)^2}{60} + \frac{(40-25)^2}{40}$$
 (M1)(A2)

Note: Award (M1) for using $\sum \frac{(f_e - f_0)^2}{f_e}$, (A2) for all values

correct.

Special case: Award (A1) if only 1 value is incorrect.

= 12.8 (AG)

(ii) 2 degrees of freedom (A1)

$$\chi^2_{2,005} = 5.991$$
 (A1)

$$\chi^2_{2,0.05} = 5.991 \tag{A}$$

Any of: (then reject H_0 , accept H_1) (R1) 6 (iii) Position is dependent on gender.

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17.	(a)	p = 25.2 $q = 16.8$ $r = 12.4$	(A1)(A1)(A1)	3
	(b)	(i) H_0 : There is no connection between gender and subj	ect taken. (C1)	
		(ii) Degrees of freedom = $(3 - 1)(2 - 1) = 2 \times 1$ = 2	(M1) (AG)	
		(iii) $\chi^2(2) = 5.99$	(A1)	3
	(c)	Accept H ₀	(C1)	
		Since 1.78 < 5.99	(R1)	2

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(A3)

(A1)

18. (a)

	Billiards	Snooker	Darts	Totals
Male Expected	32.9	16.4	13.7	63
Female Expected	27.1	13.6	11.3	52
	60	30	25	115

Note: Award (A3) for 6 correct expected values (bold), (A2) for 4 correct, (A1) for 2 correct.

H₀: Choice of game is independent of gender (A1)

H1: Choice of game is not independent of gender (A1)

Degree of freedom:
$$(3-1)(2-1) = 2$$

$$\chi^{2} = \sum \frac{(f_{0} - f_{e})^{2}}{f_{e}} = \frac{(39 - 32.9)^{2}}{32.9} + \dots$$
(M2)

= 7.77 (3 s.f.) [or 7.79 from GDC] (A1)

But
$$\chi^2_{5\%}(2) = 5.99$$
 (from table) (M1)

 $\chi^2 = 7.77 > \chi^2_{5\%}$ (2) and we do reject H₀ (A1)(R1)

Hence: Choice of game is dependent on gender. (A1) 13

(b)	(i)	The frequency for males choosing Billiards is less than 5	(R1)
<-/			

(ii) Snooker – In order to preserve the diversity of games (R1)
 OR
 Darts – it has the next smallest number of members (R1)

(c) (i)
$$\frac{31}{122}$$
 or 0.254 (3 s.f.) (A1)

(ii)
$$\frac{72}{122}$$
 or 0.590 (3 s.f.) (A1) 2

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