Chapter 6   Matrices   Exam Questions

READ THESE INSTRUCTIONS FIRST
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are reminded of the need for clear presentation in your answers.

Jun_02 P1 q.10

A toothpaste firm supplies tubes of toothpaste to 5 different stores. The number of tubes of toothpaste supplied per delivery to each store, the sizes and sale prices of the tubes, together with the number of deliveries made to each store over a 3-month period are shown in the table below.

		Number of tubes per delivery			Number of deliveries over 3 months
Size of tube		50 ml	75 ml	100 ml	
Name of store	Alwin	400	300	400	13
	Bestbuy	–	–	600	7
	Costless	400	–	600	10
	Dealwise	500	300	–	5
	Econ	600	600	400	8
Sale price per tube		$2.10	$3.00	$3.75	

(i)   Write down two matrices such that the elements of their product under matrix multiplication would give the volume of toothpaste supplied to each store per delivery.

(ii) Write down two matrices such that the elements of their product under matrix multiplication would give the number of tubes of toothpaste of each size supplied by the firm over the 3-month period. Find this product.

(iii) Using the matrix product found in part (ii) and a further matrix, find the total amount of money which would be obtained from the sale of all the tubes of toothpaste delivered over the 3-month period.

[8]

Jun_02 P2 q.1

It is given that $\mathbf{A} = \begin{pmatrix} 5 & 7 \\ 4 & 5 \end{pmatrix}$ and that $\mathbf{A} - 3\mathbf{A}^{-1} - k\mathbf{I} = \mathbf{0}$, where $\mathbf{I}$ is the identity matrix and $\mathbf{0}$ is the zero matrix. Evaluate k. [4]

A company produces 4 types of central heating radiator, known as types A, B, C and D.
A builder buys radiators for all the houses on a new estate. There are 20 small houses, 30 medium-sized houses and 15 large houses.

A small house needs 3 radiators of type A, 2 of type B and 2 of type C.
A medium-sized house needs 2 radiators of type A, 3 of type C and 3 of type D.
A large house needs 1 radiator of type B, 6 of type C and 3 of type D.

The costs of the radiators are $30 for type A, $40 for B, $50 for C and $80 for D.

Using matrix multiplication twice, find the total cost to the builder of all the radiators for the estate.   [6]

Write down the inverse of the matrix $\begin{pmatrix} 4 & 3 \\ 7 & 6 \end{pmatrix}$ and use this to solve the simultaneous equations

$$4x + 3y + 7 = 0,$$
$$7x + 6y + 16 = 0. \qquad [4]$$

Given that $\quad A = \begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix}, \quad$ find B such that $\quad 4A^{-1} + B = A^2$. [6]

A small manufacturing firm produces four types of product, A, B, C and D. Each product requires three processes – assembly, finishing and packaging. The number of minutes required for each type of product for each process and the cost, in $ per minute, of each process are given in the following table.

Process \ Type	Number of minutes				Cost per minute ($)
	A	B	C	D	
Assembly	8	6	6	5	0.60
Finishing	5	4	3	2	0.20
Packaging	3	3	2	2	0.50

The firm receives an order for 40 of type A, 50 of type B, 50 of type C and 60 of type D. Write down three matrices such that matrix multiplication will give the total cost of meeting this order. Hence evaluate this total cost.   [6]

Given $\mathbf{A} = \begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ -2 & 3 \end{pmatrix}$, write down the inverse of $\mathbf{A}$ and of $\mathbf{B}$.　　　　　[3]

Hence find

(i)　the matrix $\mathbf{C}$ such that $2\mathbf{A}^{-1} + \mathbf{C} = \mathbf{B}$,　　　　　[2]

(ii)　the matrix $\mathbf{D}$ such that $\mathbf{BD} = \mathbf{A}$.　　　　　[2]

The table below shows

the daily production, in kilograms, of two types, S_1 and S_2, of sweets from a small company,

the percentages of the ingredients A, B and C required to produce S_1 and S_2.

	Percentage			Daily production (kg)
	A	B	C	
Type S_1	60	30	10	300
Type S_2	50	40	10	240

Given that the costs, in dollars per kilogram, of A, B and C are 4, 6 and 8 respectively, use matrix
multiplication to calculate the total cost of daily production.　　　　　[6]

Given that $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ -5 & 4 \end{pmatrix}$, find $\mathbf{A}^{-1}$ and **hence** solve the simultaneous equations

$$2x + 3y + 4 = 0$$
$$-5x + 4y + 13 = 0.$$
　　　　　[4]

Given that　$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$,　find $(\mathbf{A}^2)^{-1}$.　　　　　[4]

A flower show is held over a three-day period – Thursday, Friday and Saturday. The table below shows the entry price per day for an adult and for a child, and the number of adults and children attending on each day.

	Thursday	Friday	Saturday
Price ($) – Adult	12	10	10
Price ($) – Child	5	4	4
Number of adults	300	180	400
Number of children	40	40	150

(i) Write down two matrices such that their product will give the amount of entry money paid on Thursday and hence calculate this product. [2]

(ii) Write down two matrices such that the elements of their product give the amount of entry money paid for each of Friday and Saturday and hence calculate this product. [2]

(iii) Calculate the total amount of entry money paid over the three-day period. [1]

It is given that $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 5 & p \end{pmatrix}$ and that $\mathbf{A} + \mathbf{A}^{-1} = k\mathbf{I}$, where p and k are constants and $\mathbf{I}$ is the identity matrix.
Evaluate p and k. [6]