Algebra Binomial P1 non calc MS

0 min
0 marks

1. (a) evidence of expanding M1

e.g. $2^4 + 4(2^3)x + 6(2^2)x^2 + 4(2)x^3 + x^4$, $(4 + 4x + x^2)(4 + 4x + x^2)$

$(2 + x)^4 = 16 + 32x + 24x^2 + 8x^3 + x^4$ A2 N2

(b) finding coefficients 24 and 1 (A1)(A1)

term is $25x^2$ A1 N3

[6]

2. (a) $n = 10$ A1 N1

(b) $a = p$, $b = 2q$ (or $a = 2q$, $b = p$) A1A1 N1N1

(c) $\binom{10}{5}p^5(2q)^5$ A1A1A1 N3

[6]

3. **METHOD 1**

Using binomial expansion (M1)

$\left(3+\sqrt{7}\right)^3 = 3^3 + \binom{3}{1}3^2\left(\sqrt{7}\right)+\binom{3}{2}3\left(\sqrt{7}\right)^2+\left(\sqrt{7}\right)^3$ (A1)

$= 27 + 27\sqrt{7} + 63 + 7\sqrt{7}$ (A2)

$\left(3+\sqrt{7}\right)^3 = 90+34\sqrt{7}$ (so $p = 90$, $q = 34$) (A1)(A1)(C3)(C3)

METHOD 2

For multiplying (M1)

$\left(3+\sqrt{7}\right)^2\left(3+\sqrt{7}\right) = \left(9+6\sqrt{7}+7\right)\left(3+\sqrt{7}\right)$ (A1)

$= 27 + 9\sqrt{7} + 18\sqrt{7} + 42 + 21 + 7\sqrt{7}$

$(= 27 + 27\sqrt{7} + 63 + 7\sqrt{7})$ (A2)

$\left(3+\sqrt{7}\right)^3 = 90+34\sqrt{7}$ (so $p = 90$, $q = 34$) (A1)(A1)(C3)(C3)

[6]

4. $\binom{8}{3}(2)^5(-3x)^3$ $\left(\text{Accept } \binom{8}{5}\right)$ (M1)(A1)(A1)(A1)

Term is $-48\,384x^3$ (A2) (C6)

[6]

5. Selecting one term (may be implied) (M1)

$\binom{7}{2}5^2(2x^2)^5$ (A1)(A1)(A1)

$= 16800x^{10}$ (A1)(A1) (C6)

Note: *Award C5 for 16800.*

[6]

6. $\ldots + 6 \times 2^2(ax)^2 + 4 \times 2(ax)^3 + (ax)^4$ (M1)(M1)(M1)

$= \ldots + 24a^2x^2 + 8a^3x^3 + a^4x^4$ (A1)(A1)(A1) (C6)

Notes: *Award C3 if brackets omitted, leading to $24ax^2 + 8ax^3 + ax^4$. Award C4 if correct expression with brackets as in first line of markscheme is given as final answer.*

[6]

7. (a) 10 (A2) (C2)

(b) $(3x^2)^3\left(-\frac{1}{x}\right)^6$ [for correct exponents] (M1)(A1)

$\binom{9}{6}3^3x^6\frac{1}{x^6}\left(\text{or } 84 \times 3^3x^6\frac{1}{x^6}\right)$ (A1)

constant = 2268 (A1) (C4)

[6]

8. Term involving x^3 is $\binom{5}{3}(2)^2(-x)^3$ (A1)(A1)(A1)

$\binom{5}{3} = 10$ (A1)

Therefore, term $= -40x^3$ (A1)

$\Rightarrow$ The coefficient is -40 (A1) (C6)

[6]

9. $(3x + 2y)^4 = (3x)^4 + \binom{4}{1}(3x)^2(2y) + \binom{4}{2}(3x)^2(2y)^2 + \binom{4}{3}(3x)(2y)^3 + (2y)^4$ (A1)

$= 81x^4 + 216x^3y + \mathbf{216x^2y^2 + 96xy^3 + 16y^4}$ (A1)(A1)(A1) (C4)

[4]

10. (a) $(1 + 1)^4 = 2^4 = 1 + \binom{4}{1}(1) + \binom{4}{2}(1^2) + \binom{4}{3}(1^3) + 1^4$ (M1)

$\Rightarrow \binom{4}{1} + \binom{4}{2} + \binom{4}{3} = 16 - 2$

$= 14$ (A1) (C2)

(b) $(1 + 1)^9 = 1 + \binom{9}{1} + \binom{9}{2} + \binom{9}{3} + \ldots + \binom{9}{8} + 1$ (M1)

$\Rightarrow \binom{9}{1} + \binom{9}{2} + \binom{9}{3} + \ldots + \binom{9}{8} = 2^9 - 2$

$= 510$ (A1) (C2)

[4]

11. $(a + b)^{12}$

Coefficient of a^5b^7 is $\binom{12}{5} = \binom{12}{7}$ (M1)(A1)

$= 792$ (A2) (C4)

[4]

12. Required term is $\binom{8}{5}(3x)^5(-2)^3$ (A1)(A1)(A1)

Therefore the coefficient of x^5 is $56 \times 243 \times -8$
$= -108864$ (A1) (C4)

[4]

13. $(5a + b)^7 = \ldots + \binom{7}{4}(5a)^3(b)^4 + \ldots$ (M1)

$= \frac{7\times6\times5\times4}{1\times2\times3\times4} \times 5^3 \times (a^3b^4) = 35 \times 5^3 \times a^3b^4$ (M1)(A1)

So the coefficient is 4375 (A1) (C4)

[4]