SL - Binomial Expansion Answers

0 min
0 marks

1. Required term is $\binom{8}{5}(3x)^5(-2)^3$ (A1)(A1)(A1)

Therefore the coefficient of x^5 is $56 \times 243 \times -8$
$= -108864$ (A1)

[4]

2. $(5a + b)^7 = \ldots + \binom{7}{4}(5a)^3(b)^4 + \ldots$ (M1)

$= \dfrac{7\times6\times5\times4}{1\times2\times3\times4} \times 5^3 \times (a^3b^4) = 35 \times 5^3 \times a^3b^4$ (M1)(A1)

So the coefficient is 4375 (A1)

[4]

3. $(a + b)^{12}$

Coefficient of a^5b^7 is $\binom{12}{5} = \binom{12}{7}$ (M1)(A1)

$= 792$ (A2) (C4)

[4]

4. The constant term will be the term independent of the variable x. (R1)

$\left(x - \dfrac{2}{x^2}\right)^9 = x^9 + 9x^8\left(\dfrac{-2}{x^2}\right) + \ldots + \binom{9}{3}x^6\left(\dfrac{-2}{x^2}\right)^3 + \ldots + \left(\dfrac{-2}{x^2}\right)^9$ (M1)

$\binom{9}{3}x^6\left(\dfrac{-2}{x^2}\right)^3 = 84x^6\left(\dfrac{-8}{x^6}\right)$ (A1)

$= -672$ (A1)

[4]

5. $(3x + 2y)^4 = (3x)^4 + \binom{4}{1}(3x)^2(2y) + \binom{4}{2}(3x)^2(2y)^2 + \binom{4}{3}(3x)(2y)^3 + (2y)^4$ (A1)

$= 81x^4 + 216x^3y + \mathbf{216x^2y^2 + 96xy^3 + 16y^4}$ (A1)(A1)(A1) (C4)

[4]

6. (a) $(1 + 1)^4 = 2^4 = 1 + \binom{4}{1}(1) + \binom{4}{2}(1^2) + \binom{4}{3}(1^3) + 1^4$ (M1)

$\Rightarrow \binom{4}{1} + \binom{4}{2} + \binom{4}{3} = 16 - 2$

$= 14$ (A1) (C2)

(b) $(1 + 1)^9 = 1 + \binom{9}{1} + \binom{9}{2} + \binom{9}{3} + \ldots + \binom{9}{8} + 1$ (M1)

$\Rightarrow \binom{9}{1} + \binom{9}{2} + \binom{9}{3} + \ldots + \binom{9}{8} = 2^9 - 2$

$= 510$ (A1) (C2)

[4]

7. (a) 10 (A2) (C2)

(b) $(3x^2)^3 \left(-\frac{1}{x}\right)^6$ [for correct exponents] (M1)(A1)

$\binom{9}{6} 3^3 x^6 \frac{1}{x^6} \left(\text{or } 84 \times 3^3 x^6 \frac{1}{x^6}\right)$ (A1)

constant =2268 (A1) (C4)

[6]

8. Term involving x^3 is $\binom{5}{3}(2)^2(-x)^3$ (A1)(A1)(A1)

$\binom{5}{3} = 10$ (A1)

Therefore, term = $-40x^3$ (A1)

$\Rightarrow$ The coefficient is -40 (A1) (C6)

[6]

$$x^6 \frac{1}{x^6}\left(\text{or } 84\times 3^3 x^6 \frac{1}{x^6}\right)$$

9. Selecting one term (may be implied) (M1)

$\binom{7}{2}5^2(2x^2)^5$ (A1)(A1)(A1)

$= 16\,800\, x^{10}$ (A1)(A1) (C6)

Note: *Award C5 for 16 800*

[6]

10. $... + 6 \times 2^2(ax)^2 + 4 \times 2(ax)^3 + (ax)^4$ (M1)(M1)(M1)

$= ...+ 24a^2x^2 + 8a^3x^3 + a^4x^4$ (A1)(A1)(A1) (C6)

Notes: *Award C3 if brackets omitted, leading to* $24a^2x^2 + 8a^3x^3 + a^4x^4$. *Award C4 if correct expression with brackets as in first line of markscheme is given as final answer.*

[6]

11. (a) 6 terms (A1) (C1)

(b) $\binom{5}{3} = 10,\ (-2)^3 = -8,\ \left(x^2\right)^2$ (A1)(A1)(A1)

fourth term is $-80x^4$ (A1)

for extracting the coefficient $A = -80$ (A1) (C5)

[6]

12. $\binom{8}{3}(2)^5(-3x)^3$ $\left(\text{Accept } \binom{8}{5}\right)$ (M1)(A1)(A1)(A1)

Term is $-48\,384x^3$ (A2) (C6)

[6]

13. Area of large sector $\frac{1}{2}r^2\theta = \frac{1}{2}16^2 \times 1.5$ (M1)

$= 192$ (A1)

Area of small sector $\frac{1}{2}r^2\theta = \frac{1}{2} \times 10^2 \times 1.5$ (M1)

$= 75$ (A1)

Shaded area = large area – small area = 192 – 75 (M1)

= 117 (A1) (C6)

[6]