



International Baccalaureate®
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Diploma Programme

Mathematics SL guide

First examinations 2014



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IB mission statement

The International Baccalaureate aims to develop inquiring, knowledgeable and caring young people who help to create a better and more peaceful world through intercultural understanding and respect.

To this end the organization works with schools, governments and international organizations to develop challenging programmes of international education and rigorous assessment.

These programmes encourage students across the world to become active, compassionate and lifelong learners who understand that other people, with their differences, can also be right.

IB learner profile

The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world.

IB learners strive to be:

Inquirers	They develop their natural curiosity. They acquire the skills necessary to conduct inquiry and research and show independence in learning. They actively enjoy learning and this love of learning will be sustained throughout their lives.
Knowledgeable	They explore concepts, ideas and issues that have local and global significance. In so doing, they acquire in-depth knowledge and develop understanding across a broad and balanced range of disciplines.
Thinkers	They exercise initiative in applying thinking skills critically and creatively to recognize and approach complex problems, and make reasoned, ethical decisions.
Communicators	They understand and express ideas and information confidently and creatively in more than one language and in a variety of modes of communication. They work effectively and willingly in collaboration with others.
Principled	They act with integrity and honesty, with a strong sense of fairness, justice and respect for the dignity of the individual, groups and communities. They take responsibility for their own actions and the consequences that accompany them.
Open-minded	They understand and appreciate their own cultures and personal histories, and are open to the perspectives, values and traditions of other individuals and communities. They are accustomed to seeking and evaluating a range of points of view, and are willing to grow from the experience.
Caring	They show empathy, compassion and respect towards the needs and feelings of others. They have a personal commitment to service, and act to make a positive difference to the lives of others and to the environment.
Risk-takers	They approach unfamiliar situations and uncertainty with courage and forethought, and have the independence of spirit to explore new roles, ideas and strategies. They are brave and articulate in defending their beliefs.
Balanced	They understand the importance of intellectual, physical and emotional balance to achieve personal well-being for themselves and others.
Reflective	They give thoughtful consideration to their own learning and experience. They are able to assess and understand their strengths and limitations in order to support their learning and personal development.

Syllabus outline

Syllabus component	Teaching hours
	SL
All topics are compulsory. Students must study all the sub-topics in each of the topics in the syllabus as listed in this guide. Students are also required to be familiar with the topics listed as prior learning.	
Topic 1 Algebra	9
Topic 2 Functions and equations	24
Topic 3 Circular functions and trigonometry	16
Topic 4 Vectors	16
Topic 5 Statistics and probability	35
Topic 6 Calculus	40
Mathematical exploration Internal assessment in mathematics SL is an individual exploration. This is a piece of written work that involves investigating an area of mathematics.	10
Total teaching hours	150

Prior learning topics

As noted in the previous section on prior learning, it is expected that all students have extensive previous mathematical experiences, but these will vary. It is expected that mathematics SL students will be familiar with the following topics before they take the examinations, because questions assume knowledge of them. Teachers must therefore ensure that any topics listed here that are unknown to their students at the start of the course are included at an early stage. They should also take into account the existing mathematical knowledge of their students to design an appropriate course of study for mathematics SL. This table lists the knowledge, together with the syllabus content, that is essential to successful completion of the mathematics SL course.

Students must be familiar with SI (*Système International*) units of length, mass and time, and their derived units.

Topic	Content
Number	<p>Routine use of addition, subtraction, multiplication and division, using integers, decimals and fractions, including order of operations.</p> <p>Simple positive exponents.</p> <p>Simplification of expressions involving roots (surds or radicals).</p> <p>Prime numbers and factors, including greatest common divisors and least common multiples.</p> <p>Simple applications of ratio, percentage and proportion, linked to similarity.</p> <p>Definition and elementary treatment of absolute value (modulus), a.</p> <p>Rounding, decimal approximations and significant figures, including appreciation of errors.</p> <p>Expression of numbers in standard form (scientific notation), that is, $a \times 10^k$, $1 \leq a < 10$, $k \in \mathbb{Z}$.</p>
Sets and numbers	<p>Concept and notation of sets, elements, universal (reference) set, empty (null) set, complement, subset, equality of sets, disjoint sets.</p> <p>Operations on sets: union and intersection.</p> <p>Commutative, associative and distributive properties.</p> <p>Venn diagrams.</p> <p>Number systems: natural numbers; integers, \mathbb{Z}; rationals, \mathbb{Q}, and irrationals; real numbers, \mathbb{R}.</p> <p>Intervals on the real number line using set notation and using inequalities. Expressing the solution set of a linear inequality on the number line and in set notation.</p> <p>Mappings of the elements of one set to another. Illustration by means of sets of ordered pairs, tables, diagrams and graphs.</p>
Algebra	<p>Manipulation of simple algebraic expressions involving factorization and expansion, including quadratic expressions.</p> <p>Rearrangement, evaluation and combination of simple formulae. Examples from other subject areas, particularly the sciences, should be included.</p> <p>The linear function and its graph, gradient and y-intercept.</p> <p>Addition and subtraction of algebraic fractions.</p> <p>The properties of order relations: $<$, \leq, $>$, \geq.</p> <p>Solution of equations and inequalities in one variable, including cases with rational coefficients.</p> <p>Solution of simultaneous equations in two variables.</p>

Topic	Content
Trigonometry	<p>Angle measurement in degrees. Compass directions and three figure bearings.</p> <p>Right-angle trigonometry. Simple applications for solving triangles.</p> <p>Pythagoras' theorem and its converse.</p>
Geometry	<p>Simple geometric transformations: translation, reflection, rotation, enlargement. Congruence and similarity, including the concept of scale factor of an enlargement.</p> <p>The circle, its centre and radius, area and circumference. The terms "arc", "sector", "chord", "tangent" and "segment".</p> <p>Perimeter and area of plane figures. Properties of triangles and quadrilaterals, including parallelograms, rhombuses, rectangles, squares, kites and trapeziums (trapezoids); compound shapes.</p> <p>Volumes of prisms, pyramids, spheres, cylinders and cones.</p>
Coordinate geometry	<p>Elementary geometry of the plane, including the concepts of dimension for point, line, plane and space. The equation of a line in the form $y = mx + c$.</p> <p>Parallel and perpendicular lines, including $m_1 = m_2$ and $m_1 m_2 = -1$.</p> <p>Geometry of simple plane figures.</p> <p>The Cartesian plane: ordered pairs (x, y), origin, axes.</p> <p>Mid-point of a line segment and distance between two points in the Cartesian plane and in three dimensions.</p>
Statistics and probability	<p>Descriptive statistics: collection of raw data; display of data in pictorial and diagrammatic forms, including pie charts, pictograms, stem and leaf diagrams, bar graphs and line graphs.</p> <p>Obtaining simple statistics from discrete and continuous data, including mean, median, mode, quartiles, range, interquartile range.</p> <p>Calculating probabilities of simple events.</p>

Syllabus

Syllabus content

Topic 1—Algebra

9 hours

The aim of this topic is to introduce students to some basic algebraic concepts and applications.

	Content	Further guidance	Links
1.1	<p>Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series.</p> <p>Sigma notation.</p> <p>Applications.</p>	<p>Technology may be used to generate and display sequences in several ways.</p> <p>Link to 2.6, exponential functions.</p> <p>Examples include compound interest and population growth.</p>	<p>Int: The chess legend (Sissa ibn Dahir).</p> <p>Int: Aryabhata is sometimes considered the “father of algebra”. Compare with al-Khawarizmi.</p> <p>TOK: How did Gauss add up integers from 1 to 100? Discuss the idea of mathematical intuition as the basis for formal proof.</p> <p>TOK: Debate over the validity of the notion of “infinity”: finitists such as L. Kronecker consider that “a mathematical object does not exist unless it can be constructed from natural numbers in a finite number of steps”.</p> <p>TOK: What is Zeno’s dichotomy paradox? How far can mathematical facts be from intuition?</p>

	Content	Further guidance	Links
1.2	<p>Elementary treatment of exponents and logarithms.</p> <p>Laws of exponents; laws of logarithms.</p> <p>Change of base.</p>	<p><i>Examples:</i> $16^{\frac{3}{4}} = 8$; $\frac{3}{4} = \log_{16} 8$; $\log 32 = 5 \log 2$; $(2^3)^{-4} = 2^{-12}$.</p> <p><i>Examples:</i> $\log_4 7 = \frac{\ln 7}{\ln 4}$, $\log_{25} 125 = \frac{\log_5 125}{\log_5 25} \left(= \frac{3}{2} \right)$.</p> <p>Link to 2.6, logarithmic functions.</p>	<p>Appl: Chemistry 18.1 (Calculation of pH).</p> <p>TOK: Are logarithms an invention or discovery? (This topic is an opportunity for teachers to generate reflection on “the nature of mathematics”.)</p>
1.3	<p>The binomial theorem: expansion of $(a + b)^n$, $n \in \mathbb{N}$.</p> <p>Calculation of binomial coefficients using Pascal’s triangle and $\binom{n}{r}$.</p> <p>Not required: formal treatment of permutations and formula for ${}^n P_r$.</p>	<p>Counting principles may be used in the development of the theorem.</p> <p>$\binom{n}{r}$ should be found using both the formula and technology.</p> <p><i>Example:</i> finding $\binom{6}{r}$ from inputting $y = 6^n C_r X$ and then reading coefficients from the table.</p> <p>Link to 5.8, binomial distribution.</p>	<p>Aim 8: Pascal’s triangle. Attributing the origin of a mathematical discovery to the wrong mathematician.</p> <p>Int: The so-called “Pascal’s triangle” was known in China much earlier than Pascal.</p>

Topic 2—Functions and equations

24 hours

The aims of this topic are to explore the notion of a function as a unifying theme in mathematics, and to apply functional methods to a variety of mathematical situations. It is expected that extensive use will be made of technology in both the development and the application of this topic, rather than elaborate analytical techniques. On examination papers, questions may be set requiring the graphing of functions that do not explicitly appear on the syllabus, and students may need to choose the appropriate viewing window. For those functions explicitly mentioned, questions may also be set on composition of these functions with the linear function $y = ax + b$.

	Content	Further guidance	Links
2.1	<p>Concept of function $f : x \mapsto f(x)$.</p> <p>Domain, range; image (value).</p> <p>Composite functions.</p> <p>Identity function. Inverse function f^{-1}.</p> <p>Not required: domain restriction.</p>	<p><i>Example:</i> for $x \mapsto \sqrt{2-x}$, domain is $x \leq 2$, range is $y \geq 0$.</p> <p>A graph is helpful in visualizing the range.</p> <p>$(f \circ g)(x) = f(g(x))$.</p> <p>$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$.</p> <p>On examination papers, students will only be asked to find the inverse of a <i>one-to-one</i> function.</p>	<p>Int: The development of functions, Rene Descartes (France), Gottfried Wilhelm Leibniz (Germany) and Leonhard Euler (Switzerland).</p> <p>TOK: Is zero the same as “nothing”?</p> <p>TOK: Is mathematics a formal language?</p>
2.2	<p>The graph of a function; its equation $y = f(x)$.</p> <p>Function graphing skills.</p> <p>Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes, symmetry, and consideration of domain and range.</p> <p>Use of technology to graph a variety of functions, including ones not specifically mentioned.</p> <p>The graph of $y = f^{-1}(x)$ as the reflection in the line $y = x$ of the graph of $y = f(x)$.</p>	<p>Note the difference in the command terms “draw” and “sketch”.</p> <p>An analytic approach is also expected for simple functions, including all those listed under topic 2.</p> <p>Link to 6.3, local maximum and minimum points.</p>	<p>Appl: Chemistry 11.3.1 (sketching and interpreting graphs); geographic skills.</p> <p>TOK: How accurate is a visual representation of a mathematical concept? (Limits of graphs in delivering information about functions and phenomena in general, relevance of modes of representation.)</p>

	Content	Further guidance	Links
2.3	<p>Transformations of graphs.</p> <p>Translations: $y = f(x) + b$; $y = f(x - a)$.</p> <p>Reflections (in both axes): $y = -f(x)$; $y = f(-x)$.</p> <p>Vertical stretch with scale factor p: $y = pf(x)$.</p> <p>Stretch in the x-direction with scale factor $\frac{1}{q}$: $y = f(qx)$.</p> <p>Composite transformations.</p>	<p>Technology should be used to investigate these transformations.</p> <p>Translation by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ denotes horizontal shift of 3 units to the right, and vertical shift of 2 down.</p> <p><i>Example:</i> $y = x^2$ used to obtain $y = 3x^2 + 2$ by a stretch of scale factor 3 in the y-direction followed by a translation of $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.</p>	<p>Appl: Economics 1.1 (shifting of supply and demand curves).</p>
2.4	<p>The quadratic function $x \mapsto ax^2 + bx + c$: its graph, y-intercept $(0, c)$. Axis of symmetry.</p> <p>The form $x \mapsto a(x - p)(x - q)$, x-intercepts $(p, 0)$ and $(q, 0)$.</p> <p>The form $x \mapsto a(x - h)^2 + k$, vertex (h, k) .</p>	<p>Candidates are expected to be able to change from one form to another.</p> <p>Links to 2.3, transformations; 2.7, quadratic equations.</p>	<p>Appl: Chemistry 17.2 (equilibrium law).</p> <p>Appl: Physics 2.1 (kinematics).</p> <p>Appl: Physics 4.2 (simple harmonic motion).</p> <p>Appl: Physics 9.1 (HL only) (projectile motion).</p>

	Content	Further guidance	Links
2.5	<p>The reciprocal function $x \mapsto \frac{1}{x}$, $x \neq 0$: its graph and self-inverse nature.</p> <p>The rational function $x \mapsto \frac{ax+b}{cx+d}$ and its graph.</p> <p>Vertical and horizontal asymptotes.</p>	<p>Examples: $h(x) = \frac{4}{3x-2}$, $x \neq \frac{2}{3}$; $y = \frac{x+7}{2x-5}$, $x \neq \frac{5}{2}$.</p> <p>Diagrams should include all asymptotes and intercepts.</p>	
2.6	<p>Exponential functions and their graphs: $x \mapsto a^x$, $a > 0$, $x \mapsto e^x$.</p> <p>Logarithmic functions and their graphs: $x \mapsto \log_a x$, $x > 0$, $x \mapsto \ln x$, $x > 0$.</p> <p>Relationships between these functions: $a^x = e^{x \ln a}$; $\log_a a^x = x$; $a^{\log_a x} = x$, $x > 0$.</p>	<p>Links to 1.1, geometric sequences; 1.2, laws of exponents and logarithms; 2.1, inverse functions; 2.2, graphs of inverses; and 6.1, limits.</p>	<p>Int: The Babylonian method of multiplication: $ab = \frac{(a+b)^2 - a^2 - b^2}{2}$. Sulba Sutras in ancient India and the Bakhshali Manuscript contained an algebraic formula for solving quadratic equations.</p>

	Content	Further guidance	Links
2.7	<p>Solving equations, both graphically and analytically.</p> <p>Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach.</p> <p>Solving $ax^2 + bx + c = 0$, $a \neq 0$.</p> <p>The quadratic formula.</p> <p>The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots.</p> <p>Solving exponential equations.</p>	<p>Solutions may be referred to as roots of equations or zeros of functions.</p> <p>Links to 2.2, function graphing skills; and 2.3–2.6, equations involving specific functions.</p> <p><i>Examples:</i> $e^x = \sin x$, $x^4 + 5x - 6 = 0$.</p> <p><i>Example:</i> Find k given that the equation $3kx^2 + 2x + k = 0$ has two equal real roots.</p> <p><i>Examples:</i> $2^{x-1} = 10$, $\left(\frac{1}{3}\right)^x = 9^{x+1}$.</p> <p>Link to 1.2, exponents and logarithms.</p>	
2.8	Applications of graphing skills and solving equations that relate to real-life situations.	Link to 1.1, geometric series.	<p>Appl: Compound interest, growth and decay; projectile motion; braking distance; electrical circuits.</p> <p>Appl: Physics 7.2.7–7.2.9, 13.2.5, 13.2.6, 13.2.8 (radioactive decay and half-life)</p>

Topic 3—Circular functions and trigonometry

16 hours

The aims of this topic are to explore the circular functions and to solve problems using trigonometry. On examination papers, radian measure should be assumed unless otherwise indicated.

	Content	Further guidance	Links
3.1	The circle: radian measure of angles; length of an arc; area of a sector.	Radian measure may be expressed as exact multiples of π , or decimals.	<p>Int: Seki Takakazu calculating π to ten decimal places.</p> <p>Int: Hipparchus, Menelaus and Ptolemy.</p> <p>Int: Why are there 360 degrees in a complete turn? Links to Babylonian mathematics.</p> <p>TOK: Which is a better measure of angle: radian or degree? What are the “best” criteria by which to decide?</p> <p>TOK: Euclid’s axioms as the building blocks of Euclidean geometry. Link to non-Euclidean geometry.</p>
3.2	<p>Definition of $\cos \theta$ and $\sin \theta$ in terms of the unit circle.</p> <p>Definition of $\tan \theta$ as $\frac{\sin \theta}{\cos \theta}$.</p> <p>Exact values of trigonometric ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples.</p>	<p>The equation of a straight line through the origin is $y = x \tan \theta$.</p> <p><i>Examples:</i></p> $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}, \cos \frac{3\pi}{4} = -\frac{1}{\sqrt{2}}, \tan 210^\circ = \frac{\sqrt{3}}{3}.$	<p>Aim 8: Who really invented “Pythagoras’ theorem”?</p> <p>Int: The first work to refer explicitly to the sine as a function of an angle is the Aryabhatiya of Aryabhata (ca. 510).</p> <p>TOK: Trigonometry was developed by successive civilizations and cultures. How is mathematical knowledge considered from a sociocultural perspective?</p>

	Content	Further guidance	Links
3.3	<p>The Pythagorean identity $\cos^2 \theta + \sin^2 \theta = 1$.</p> <p>Double angle identities for sine and cosine.</p> <p>Relationship between trigonometric ratios.</p>	<p>Simple geometrical diagrams and/or technology may be used to illustrate the double angle formulae (and other trigonometric identities).</p> <p><i>Examples:</i></p> <p>Given $\sin \theta$, finding possible values of $\tan \theta$ without finding θ.</p> <p>Given $\cos x = \frac{3}{4}$, and x is acute, find $\sin 2x$ without finding x.</p>	
3.4	<p>The circular functions $\sin x$, $\cos x$ and $\tan x$: their domains and ranges; amplitude, their periodic nature; and their graphs.</p> <p>Composite functions of the form $f(x) = a \sin(b(x+c)) + d$.</p> <p>Transformations.</p> <p>Applications.</p>	<p><i>Examples:</i></p> <p>$f(x) = \tan\left(x - \frac{\pi}{4}\right)$, $f(x) = 2 \cos(3(x-4)) + 1$.</p> <p><i>Example:</i> $y = \sin x$ used to obtain $y = 3 \sin 2x$ by a stretch of scale factor 3 in the y-direction and a stretch of scale factor $\frac{1}{2}$ in the x-direction.</p> <p>Link to 2.3, transformation of graphs.</p> <p>Examples include height of tide, motion of a Ferris wheel.</p>	Appl: Physics 4.2 (simple harmonic motion).

	Content	Further guidance	Links
3.5	<p>Solving trigonometric equations in a finite interval, both graphically and analytically.</p> <p>Equations leading to quadratic equations in $\sin x$, $\cos x$ or $\tan x$.</p> <p>Not required: the general solution of trigonometric equations.</p>	<p><i>Examples:</i> $2\sin x = 1$, $0 \leq x \leq 2\pi$, $2\sin 2x = 3\cos x$, $0^\circ \leq x \leq 180^\circ$, $2\tan(3(x-4)) = 1$, $-\pi \leq x \leq 3\pi$.</p> <p><i>Examples:</i> $2\sin^2 x + 5\cos x + 1 = 0$ for $0 \leq x < 4\pi$, $2\sin x = \cos 2x$, $-\pi \leq x \leq \pi$.</p>	
3.6	<p>Solution of triangles.</p> <p>The cosine rule.</p> <p>The sine rule, including the ambiguous case.</p> <p>Area of a triangle, $\frac{1}{2}ab\sin C$.</p> <p>Applications.</p>	<p>Pythagoras' theorem is a special case of the cosine rule.</p> <p>Link with 4.2, scalar product, noting that: $\mathbf{c} = \mathbf{a} - \mathbf{b} \Rightarrow \mathbf{c} ^2 = \mathbf{a} ^2 + \mathbf{b} ^2 - 2\mathbf{a} \cdot \mathbf{b}$.</p> <p>Examples include navigation, problems in two and three dimensions, including angles of elevation and depression.</p>	<p>Aim 8: Attributing the origin of a mathematical discovery to the wrong mathematician.</p> <p>Int: Cosine rule: Al-Kashi and Pythagoras.</p> <p>TOK: Non-Euclidean geometry: angle sum on a globe greater than 180°.</p>

Topic 4—Vectors

16 hours

The aim of this topic is to provide an elementary introduction to vectors, including both algebraic and geometric approaches. The use of dynamic geometry software is extremely helpful to visualize situations in three dimensions.

	Content	Further guidance	Links
4.1	<p>Vectors as displacements in the plane and in three dimensions.</p> <p>Components of a vector; column representation; $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$.</p> <p>Algebraic and geometric approaches to the following:</p> <ul style="list-style-type: none"> the sum and difference of two vectors; the zero vector, the vector $-\mathbf{v}$; multiplication by a scalar, $k\mathbf{v}$; parallel vectors; magnitude of a vector, \mathbf{v}; unit vectors; base vectors; \mathbf{i}, \mathbf{j} and \mathbf{k}; position vectors $\vec{OA} = \mathbf{a}$; $\vec{AB} = \vec{OB} - \vec{OA} = \mathbf{b} - \mathbf{a}$. 	<p>Link to three-dimensional geometry, x, y and z-axes.</p> <p>Components are with respect to the unit vectors \mathbf{i}, \mathbf{j} and \mathbf{k} (standard basis).</p> <p>Applications to simple geometric figures are essential.</p> <p>The difference of \mathbf{v} and \mathbf{w} is $\mathbf{v} - \mathbf{w} = \mathbf{v} + (-\mathbf{w})$. Vector sums and differences can be represented by the diagonals of a parallelogram.</p> <p>Multiplication by a scalar can be illustrated by enlargement.</p> <p>Distance between points A and B is the magnitude of \vec{AB}.</p>	<p>Appl: Physics 1.3.2 (vector sums and differences) Physics 2.2.2, 2.2.3 (vector resultants).</p> <p>TOK: How do we relate a theory to the author? Who developed vector analysis: JW Gibbs or O Heaviside?</p>

	Content	Further guidance	Links
4.2	<p>The scalar product of two vectors.</p> <p>Perpendicular vectors; parallel vectors.</p> <p>The angle between two vectors.</p>	<p>The scalar product is also known as the “dot product”.</p> <p>Link to 3.6, cosine rule.</p> <p>For non-zero vectors, $\mathbf{v} \cdot \mathbf{w} = 0$ is equivalent to the vectors being perpendicular.</p> <p>For parallel vectors, $\mathbf{w} = k\mathbf{v}$, $\mathbf{v} \cdot \mathbf{w} = \mathbf{v} \mathbf{w}$.</p>	
4.3	<p>Vector equation of a line in two and three dimensions: $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.</p> <p>The angle between two lines.</p>	<p>Relevance of \mathbf{a} (position) and \mathbf{b} (direction).</p> <p>Interpretation of t as time and \mathbf{b} as velocity, with \mathbf{b} representing speed.</p>	<p>Aim 8: Vector theory is used for tracking displacement of objects, including for peaceful and harmful purposes.</p> <p>TOK: Are algebra and geometry two separate domains of knowledge? (Vector algebra is a good opportunity to discuss how geometrical properties are described and generalized by algebraic methods.)</p>
4.4	<p>Distinguishing between coincident and parallel lines.</p> <p>Finding the point of intersection of two lines.</p> <p>Determining whether two lines intersect.</p>		

Topic 5—Statistics and probability

35 hours

The aim of this topic is to introduce basic concepts. It is expected that most of the calculations required will be done using technology, but explanations of calculations by hand may enhance understanding. The emphasis is on understanding and interpreting the results obtained, in context. Statistical tables will no longer be allowed in examinations. While many of the calculations required in examinations are estimates, it is likely that the command terms “write down”, “find” and “calculate” will be used.

	Content	Further guidance	Links
5.1	<p>Concepts of population, sample, random sample, discrete and continuous data.</p> <p>Presentation of data: frequency distributions (tables); frequency histograms with equal class intervals;</p> <p>box-and-whisker plots; outliers.</p> <p>Grouped data: use of mid-interval values for calculations; interval width; upper and lower interval boundaries; modal class.</p> <p>Not required: frequency density histograms.</p>	<p>Continuous and discrete data.</p> <p>Outlier is defined as more than $1.5 \times \text{IQR}$ from the nearest quartile.</p> <p>Technology may be used to produce histograms and box-and-whisker plots.</p>	<p>Appl: Psychology: descriptive statistics, random sample (various places in the guide).</p> <p>Aim 8: Misleading statistics.</p> <p>Int: The St Petersburg paradox, Chebychev, Pavlovsky.</p>

	Content	Further guidance	Links
5.2	<p>Statistical measures and their interpretations.</p> <p>Central tendency: mean, median, mode.</p> <p>Quartiles, percentiles.</p> <p>Dispersion: range, interquartile range, variance, standard deviation.</p> <p>Effect of constant changes to the original data.</p> <p>Applications.</p>	<p>On examination papers, data will be treated as the population.</p> <p>Calculation of mean using formula and technology. Students should use mid-interval values to estimate the mean of grouped data.</p> <p>Calculation of standard deviation/variance using only technology.</p> <p>Link to 2.3, transformations.</p> <p><i>Examples:</i></p> <p>If 5 is subtracted from all the data items, then the mean is decreased by 5, but the standard deviation is unchanged.</p> <p>If all the data items are doubled, the median is doubled, but the variance is increased by a factor of 4.</p>	<p>Appl: Psychology: descriptive statistics (various places in the guide).</p> <p>Appl: Statistical calculations to show patterns and changes; geographic skills; statistical graphs.</p> <p>Appl: Biology 1.1.2 (calculating mean and standard deviation); Biology 1.1.4 (comparing means and spreads between two or more samples).</p> <p>Int: Discussion of the different formulae for variance.</p> <p>TOK: Do different measures of central tendency express different properties of the data? Are these measures invented or discovered? Could mathematics make alternative, equally true, formulae? What does this tell us about mathematical truths?</p> <p>TOK: How easy is it to lie with statistics?</p>
5.3	<p>Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles.</p>	<p>Values of the median and quartiles produced by technology may be different from those obtained from a cumulative frequency graph.</p>	

	Content	Further guidance	Links
5.4	<p>Linear correlation of bivariate data.</p> <p>Pearson's product-moment correlation coefficient r.</p> <p>Scatter diagrams; lines of best fit.</p> <p>Equation of the regression line of y on x.</p> <p>Use of the equation for prediction purposes.</p> <p>Mathematical and contextual interpretation.</p> <p>Not required: the coefficient of determination R^2.</p>	<p>Independent variable x, dependent variable y.</p> <p>Technology should be used to calculate r. However, hand calculations of r may enhance understanding.</p> <p>Positive, zero, negative; strong, weak, no correlation.</p> <p>The line of best fit passes through the mean point.</p> <p>Technology should be used find the equation.</p> <p>Interpolation, extrapolation.</p>	<p>Appl: Chemistry 11.3.3 (curves of best fit).</p> <p>Appl: Geography (geographic skills).</p> <p>Measures of correlation; geographic skills.</p> <p>Appl: Biology 1.1.6 (correlation does not imply causation).</p> <p>TOK: Can we predict the value of x from y, using this equation?</p> <p>TOK: Can all data be modelled by a (known) mathematical function? Consider the reliability and validity of mathematical models in describing real-life phenomena.</p>
5.5	<p>Concepts of trial, outcome, equally likely outcomes, sample space (U) and event.</p> <p>The probability of an event A is $P(A) = \frac{n(A)}{n(U)}$.</p> <p>The complementary events A and A' (not A).</p> <p>Use of Venn diagrams, tree diagrams and tables of outcomes.</p>	<p>The sample space can be represented diagrammatically in many ways.</p> <p>Experiments using coins, dice, cards and so on, can enhance understanding of the distinction between (experimental) relative frequency and (theoretical) probability.</p> <p>Simulations may be used to enhance this topic.</p> <p>Links to 5.1, frequency; 5.3, cumulative frequency.</p>	<p>TOK: To what extent does mathematics offer models of real life? Is there always a function to model data behaviour?</p>

	Content	Further guidance	Links
5.6	<p>Combined events, $P(A \cup B)$.</p> <p>Mutually exclusive events: $P(A \cap B) = 0$.</p> <p>Conditional probability; the definition</p> $P(A B) = \frac{P(A \cap B)}{P(B)}.$ <p>Independent events; the definition</p> $P(A B) = P(A) = P(A B').$ <p>Probabilities with and without replacement.</p>	<p>The non-exclusivity of “or”.</p> <p>Problems are often best solved with the aid of a Venn diagram or tree diagram, without explicit use of formulae.</p>	<p>Aim 8: The gambling issue: use of probability in casinos. Could or should mathematics help increase incomes in gambling?</p> <p>TOK: Is mathematics useful to measure risks?</p> <p>TOK: Can gambling be considered as an application of mathematics? (This is a good opportunity to generate a debate on the nature, role and ethics of mathematics regarding its applications.)</p>
5.7	<p>Concept of discrete random variables and their probability distributions.</p> <p>Expected value (mean), $E(X)$ for discrete data.</p> <p>Applications.</p>	<p>Simple examples only, such as:</p> $P(X = x) = \frac{1}{18}(4 + x) \text{ for } x \in \{1, 2, 3\};$ $P(X = x) = \frac{5}{18}, \frac{6}{18}, \frac{7}{18}.$ <p>$E(X) = 0$ indicates a fair game where X represents the gain of one of the players.</p> <p>Examples include games of chance.</p>	

	Content	Further guidance	Links
5.8	<p>Binomial distribution.</p> <p>Mean and variance of the binomial distribution.</p> <p>Not required: formal proof of mean and variance.</p>	<p>Link to 1.3, binomial theorem.</p> <p>Conditions under which random variables have this distribution.</p> <p>Technology is usually the best way of calculating binomial probabilities.</p>	
5.9	<p>Normal distributions and curves.</p> <p>Standardization of normal variables (z-values, z-scores).</p> <p>Properties of the normal distribution.</p>	<p>Probabilities and values of the variable must be found using technology.</p> <p>Link to 2.3, transformations.</p> <p>The standardized value (z) gives the number of standard deviations from the mean.</p>	<p>Appl: Biology 1.1.3 (links to normal distribution).</p> <p>Appl: Psychology: descriptive statistics (various places in the guide).</p>

Topic 6—Calculus

40 hours

The aim of this topic is to introduce students to the basic concepts and techniques of differential and integral calculus and their applications.

	Content	Further guidance	Links
6.1	<p>Informal ideas of limit and convergence.</p> <p>Limit notation.</p> <p>Definition of derivative from first principles as $f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$.</p> <p>Derivative interpreted as gradient function and as rate of change.</p> <p>Tangents and normals, and their equations.</p> <p>Not required: analytic methods of calculating limits.</p>	<p><i>Example:</i> 0.3, 0.33, 0.333, ... converges to $\frac{1}{3}$.</p> <p>Technology should be used to explore ideas of limits, numerically and graphically.</p> <p><i>Example:</i> $\lim_{x \rightarrow \infty} \left(\frac{2x+3}{x-1} \right)$</p> <p>Links to 1.1, infinite geometric series; 2.5–2.7, rational and exponential functions, and asymptotes.</p> <p>Use of this definition for derivatives of simple polynomial functions only.</p> <p>Technology could be used to illustrate other derivatives.</p> <p>Link to 1.3, binomial theorem.</p> <p>Use of both forms of notation, $\frac{dy}{dx}$ and $f'(x)$, for the first derivative.</p> <p>Identifying intervals on which functions are increasing or decreasing.</p> <p>Use of both analytic approaches and technology.</p> <p>Technology can be used to explore graphs and their derivatives.</p>	<p>Appl: Economics 1.5 (marginal cost, marginal revenue, marginal profit).</p> <p>Appl: Chemistry 11.3.4 (interpreting the gradient of a curve).</p> <p>Aim 8: The debate over whether Newton or Leibnitz discovered certain calculus concepts.</p> <p>TOK: What value does the knowledge of limits have? Is infinitesimal behaviour applicable to real life?</p> <p>TOK: Opportunities for discussing hypothesis formation and testing, and then the formal proof can be tackled by comparing certain cases, through an investigative approach.</p>

	Content	Further guidance	Links
6.2	<p>Derivative of x^n ($n \in \mathbb{Q}$), $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$.</p> <p>Differentiation of a sum and a real multiple of these functions.</p> <p>The chain rule for composite functions.</p> <p>The product and quotient rules.</p> <p>The second derivative.</p> <p>Extension to higher derivatives.</p>	<p>Link to 2.1, composition of functions.</p> <p>Technology may be used to investigate the chain rule.</p> <p>Use of both forms of notation, $\frac{d^2y}{dx^2}$ and $f''(x)$.</p> <p>$\frac{d^n y}{dx^n}$ and $f^{(n)}(x)$.</p>	

	Content	Further guidance	Links
6.3	<p>Local maximum and minimum points. Testing for maximum or minimum.</p> <p>Points of inflexion with zero and non-zero gradients.</p> <p>Graphical behaviour of functions, including the relationship between the graphs of f, f' and f''. Optimization.</p> <p>Applications.</p> <p>Not required: points of inflexion where $f''(x)$ is not defined: for example, $y = x^{1/3}$ at $(0,0)$.</p>	<p>Using change of sign of the first derivative and using sign of the second derivative.</p> <p>Use of the terms “concave-up” for $f''(x) > 0$, and “concave-down” for $f''(x) < 0$.</p> <p>At a point of inflexion, $f''(x) = 0$ and changes sign (concavity change). $f''(x) = 0$ is not a sufficient condition for a point of inflexion: for example, $y = x^4$ at $(0,0)$.</p> <p>Both “global” (for large x) and “local” behaviour.</p> <p>Technology can display the graph of a derivative without explicitly finding an expression for the derivative.</p> <p>Use of the first or second derivative test to justify maximum and/or minimum values.</p> <p>Examples include profit, area, volume.</p> <p>Link to 2.2, graphing functions.</p>	<p>Appl: profit, area, volume.</p>

	Content	Further guidance	Links
6.4	<p>Indefinite integration as anti-differentiation.</p> <p>Indefinite integral of x^n ($n \in \mathbb{Q}$), $\sin x$, $\cos x$, $\frac{1}{x}$ and e^x.</p> <p>The composites of any of these with the linear function $ax + b$.</p> <p>Integration by inspection, or substitution of the form $\int f(g(x))g'(x)dx$.</p>	<p>$\int \frac{1}{x} dx = \ln x + C$, $x > 0$.</p> <p><i>Example:</i></p> $f'(x) = \cos(2x + 3) \Rightarrow f(x) = \frac{1}{2} \sin(2x + 3) + C.$ <p><i>Examples:</i></p> $\int 2x(x^2 + 1)^4 dx, \quad \int x \sin x^2 dx, \quad \int \frac{\sin x}{\cos x} dx.$	
6.5	<p>Anti-differentiation with a boundary condition to determine the constant term.</p> <p>Definite integrals, both analytically and using technology.</p> <p>Areas under curves (between the curve and the x-axis).</p> <p>Areas between curves.</p> <p>Volumes of revolution about the x-axis.</p>	<p><i>Example:</i></p> <p>if $\frac{dy}{dx} = 3x^2 + x$ and $y = 10$ when $x = 0$, then</p> $y = x^3 + \frac{1}{2}x^2 + 10.$ <p>$\int_a^b g'(x)dx = g(b) - g(a)$.</p> <p>The value of some definite integrals can only be found using technology.</p> <p>Students are expected to first write a correct expression before calculating the area.</p> <p>Technology may be used to enhance understanding of area and volume.</p>	<p>Int: Successful calculation of the volume of the pyramidal frustum by ancient Egyptians (Egyptian Moscow papyrus).</p> <p>Use of infinitesimals by Greek geometers.</p> <p>Accurate calculation of the volume of a cylinder by Chinese mathematician Liu Hui</p> <p>Int: Ibn Al Haytham: first mathematician to calculate the integral of a function, in order to find the volume of a paraboloid.</p>
6.6	<p>Kinematic problems involving displacement s, velocity v and acceleration a.</p> <p>Total distance travelled.</p>	<p>$v = \frac{ds}{dt}$; $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$.</p> <p>Total distance travelled $= \int_{t_1}^{t_2} v dt$.</p>	<p>Appl: Physics 2.1 (kinematics).</p>

Glossary of command terms

Command terms with definitions

Students should be familiar with the following key terms and phrases used in examination questions, which are to be understood as described below. Although these terms will be used in examination questions, other terms may be used to direct students to present an argument in a specific way.

Calculate	Obtain a numerical answer showing the relevant stages in the working.
Comment	Give a judgment based on a given statement or result of a calculation.
Compare	Give an account of the similarities between two (or more) items or situations, referring to both (all) of them throughout.
Compare and contrast	Give an account of the similarities and differences between two (or more) items or situations, referring to both (all) of them throughout.
Construct	Display information in a diagrammatic or logical form.
Contrast	Give an account of the differences between two (or more) items or situations, referring to both (all) of them throughout.
Deduce	Reach a conclusion from the information given.
Demonstrate	Make clear by reasoning or evidence, illustrating with examples or practical application.
Describe	Give a detailed account.
Determine	Obtain the only possible answer.
Differentiate	Obtain the derivative of a function.
Distinguish	Make clear the differences between two or more concepts or items.
Draw	Represent by means of a labelled, accurate diagram or graph, using a pencil. A ruler (straight edge) should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted (if appropriate) and joined in a straight line or smooth curve.
Estimate	Obtain an approximate value.
Explain	Give a detailed account, including reasons or causes.
Find	Obtain an answer, showing relevant stages in the working.
Hence	Use the preceding work to obtain the required result.
Hence or otherwise	It is suggested that the preceding work is used, but other methods could also receive credit.
Identify	Provide an answer from a number of possibilities.

Integrate	Obtain the integral of a function.
Interpret	Use knowledge and understanding to recognize trends and draw conclusions from given information.
Investigate	Observe, study, or make a detailed and systematic examination, in order to establish facts and reach new conclusions.
Justify	Give valid reasons or evidence to support an answer or conclusion.
Label	Add labels to a diagram.
List	Give a sequence of brief answers with no explanation.
Plot	Mark the position of points on a diagram.
Predict	Give an expected result.
Show	Give the steps in a calculation or derivation.
Show that	Obtain the required result (possibly using information given) without the formality of proof. “Show that” questions do not generally require the use of a calculator.
Sketch	Represent by means of a diagram or graph (labelled as appropriate). The sketch should give a general idea of the required shape or relationship, and should include relevant features.
Solve	Obtain the answer(s) using algebraic and/or numerical and/or graphical methods.
State	Give a specific name, value or other brief answer without explanation or calculation.
Suggest	Propose a solution, hypothesis or other possible answer.
Verify	Provide evidence that validates the result.
Write down	Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown.

Notation list

Of the various notations in use, the IB has chosen to adopt a system of notation based on the recommendations of the International Organization for Standardization (ISO). This notation is used in the examination papers for this course without explanation. If forms of notation other than those listed in this guide are used on a particular examination paper, they are defined within the question in which they appear.

Because students are required to recognize, though not necessarily use, IB notation in examinations, it is recommended that teachers introduce students to this notation at the earliest opportunity. Students are **not** allowed access to information about this notation in the examinations.

Students must always use correct mathematical notation, not calculator notation.

\mathbb{N}	the set of positive integers and zero, $\{0, 1, 2, 3, \dots\}$
\mathbb{Z}	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
\mathbb{Z}^+	the set of positive integers, $\{1, 2, 3, \dots\}$
\mathbb{Q}	the set of rational numbers
\mathbb{Q}^+	the set of positive rational numbers, $\{x \mid x \in \mathbb{Q}, x > 0\}$
\mathbb{R}	the set of real numbers
\mathbb{R}^+	the set of positive real numbers, $\{x \mid x \in \mathbb{R}, x > 0\}$
$\{x_1, x_2, \dots\}$	the set with elements x_1, x_2, \dots
$n(A)$	the number of elements in the finite set A
$\{x \mid \}$	the set of all x such that
\in	is an element of
\notin	is not an element of
\emptyset	the empty (null) set
U	the universal set
\cup	Union

\cap	Intersection
\subset	is a proper subset of
\subseteq	is a subset of
A'	the complement of the set A
$a b$	a divides b
$a^{1/n}, \sqrt[n]{a}$	a to the power of $\frac{1}{n}$, n^{th} root of a (if $a \geq 0$ then $\sqrt[n]{a} \geq 0$)
$ x $	modulus or absolute value of x , that is $\begin{cases} x & \text{for } x \geq 0, x \in \mathbb{R} \\ -x & \text{for } x < 0, x \in \mathbb{R} \end{cases}$
\approx	is approximately equal to
$>$	is greater than
\geq	is greater than or equal to
$<$	is less than
\leq	is less than or equal to
\nlessgtr	is not greater than
\nlessgtr	is not less than
u_n	the n^{th} term of a sequence or series
d	the common difference of an arithmetic sequence
r	the common ratio of a geometric sequence
S_n	the sum of the first n terms of a sequence, $u_1 + u_2 + \dots + u_n$
S_∞	the sum to infinity of a sequence, $u_1 + u_2 + \dots$
$\sum_{i=1}^n u_i$	$u_1 + u_2 + \dots + u_n$
$\binom{n}{r}$	the r^{th} binomial coefficient, $r = 0, 1, 2, \dots$, in the expansion of $(a + b)^n$
$f : A \rightarrow B$	f is a function under which each element of set A has an image in set B
$f : x \mapsto y$	f is a function under which x is mapped to y

$f(x)$	the image of x under the function f
f^{-1}	the inverse function of the function f
$f \circ g$	the composite function of f and g
$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as x tends to a
$\frac{dy}{dx}$	the derivative of y with respect to x
$f'(x)$	the derivative of $f(x)$ with respect to x
$\frac{d^2 y}{dx^2}$	the second derivative of y with respect to x
$f''(x)$	the second derivative of $f(x)$ with respect to x
$\frac{d^n y}{dx^n}$	the n^{th} derivative of y with respect to x
$f^{(n)}(x)$	the n^{th} derivative of $f(x)$ with respect to x
$\int y \, dx$	the indefinite integral of y with respect to x
$\int_a^b y \, dx$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
e^x	exponential function (base e) of x
$\log_a x$	logarithm to the base a of x
$\ln x$	the natural logarithm of x , $\log_e x$
\sin, \cos, \tan	the circular functions
$A(x, y)$	the point A in the plane with Cartesian coordinates x and y
$[AB]$	the line segment with end points A and B
AB	the length of $[AB]$
(AB)	the line containing points A and B
\hat{A}	the angle at A
\hat{CAB}	the angle between $[CA]$ and $[AB]$

$\triangle ABC$	the triangle whose vertices are A , B and C
\mathbf{v}	the vector \mathbf{v}
\vec{AB}	the vector represented in magnitude and direction by the directed line segment from A to B
\mathbf{a}	the position vector \vec{OA}
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the Cartesian coordinate axes
$ \mathbf{a} $	the magnitude of \mathbf{a}
$ \vec{AB} $	the magnitude of \vec{AB}
$\mathbf{v} \cdot \mathbf{w}$	the scalar product of \mathbf{v} and \mathbf{w}
$P(A)$	probability of event A
$P(A')$	probability of the event “not A ”
$P(A B)$	probability of the event A given the event B
x_1, x_2, \dots	Observations
f_1, f_2, \dots	frequencies with which the observations x_1, x_2, \dots occur
$\binom{n}{r}$	number of ways of selecting r items from n items
$B(n, p)$	binomial distribution with parameters n and p
$N(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
$X \sim B(n, p)$	the random variable X has a binomial distribution with parameters n and p
$X \sim N(\mu, \sigma^2)$	the random variable X has a normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
\bar{x}	mean of a set of data, x_1, x_2, x_3, \dots

z	standardized normal random variable, $z = \frac{x - \mu}{\sigma}$
Φ	cumulative distribution function of the standardized normal variable with distribution $N(0, 1)$
r	Pearson's product–moment correlation coefficient