



**MATHEMATICAL STUDIES  
STANDARD LEVEL  
PAPER 1**

Thursday 4 November 2010 (afternoon)

1 hour 30 minutes

Candidate session number

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**INSTRUCTIONS TO CANDIDATES**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.



Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. Consider the following four numbers.

$$p = 0.00314 ; q = 0.00314 \times 10^2 ; r = \frac{\pi}{1000} ; s = 3.14 \times 10^{-2}$$

- (a) One of these numbers is written in the form  $a \times 10^k$  where  $1 \leq a < 10$  and  $k \in \mathbb{Z}$ . Write down this number. [1 mark]
- (b) Write down the smallest of these numbers. [2 marks]
- (c) Write down the value of  $q + s$ . [1 mark]
- (d) Give your answer to part (c) in the form  $a \times 10^k$  where  $1 \leq a < 10$  and  $k \in \mathbb{Z}$ . [2 marks]

Working:

Answers:

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_
- (d) \_\_\_\_\_



2. (a) Complete the following truth table.

| $p$ | $q$ | ..... | $p \Rightarrow \neg q$ |
|-----|-----|-------|------------------------|
| T   | T   | F     | .....                  |
| T   | F   | T     | .....                  |
| F   | T   | F     | .....                  |
| F   | F   | T     | .....                  |

[2 marks]

Consider the propositions

$p$ : Cristina understands logic

$q$ : Cristina will do well on the logic test.

(b) Write down the following compound proposition in symbolic form.

“If Cristina understands logic then she will do well on the logic test”

[2 marks]

(c) Write down in words the contrapositive of the proposition given in part (b).

[2 marks]

Working:

Answers:

(b) \_\_\_\_\_

(c) \_\_\_\_\_



3. In a television show there is a transparent box completely filled with identical cubes. Participants have to estimate the number of cubes in the box. The box is 50 cm wide, 100 cm long and 40 cm tall.

(a) Find the volume of the box. *[2 marks]*

Joaquin estimates the volume of one cube to be  $500 \text{ cm}^3$ . He uses this value to estimate the number of cubes in the box.

(b) Find Joaquin's estimated number of cubes in the box. *[2 marks]*

The actual number of cubes in the box is 350.

(c) Find the percentage error in Joaquin's estimated number of cubes in the box. *[2 marks]*

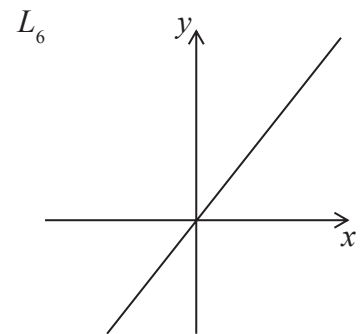
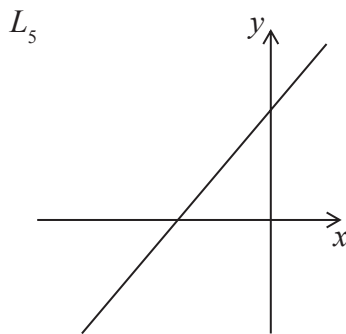
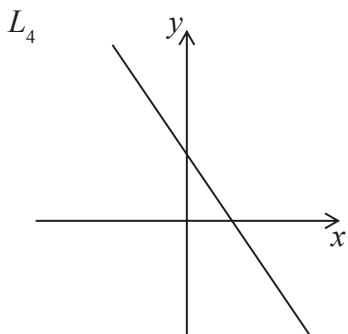
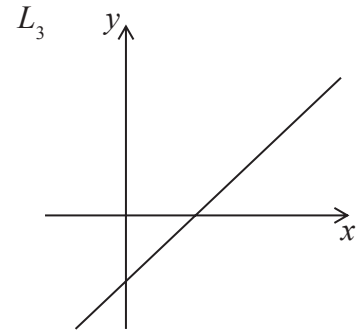
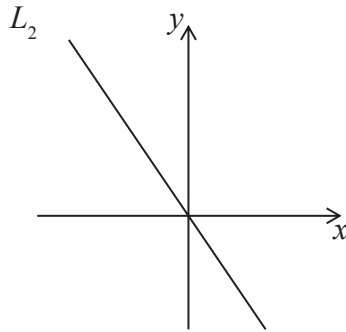
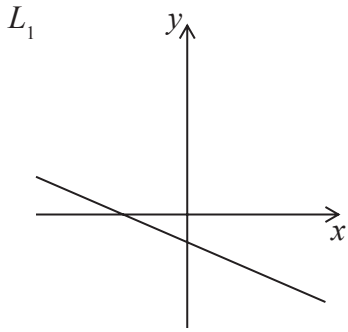
*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_



4. The following diagrams show six lines with equations of the form  $y = mx + c$ .



In the table below there are four possible conditions for the pair of values  $m$  and  $c$ .  
Match each of the given conditions with one of the lines drawn above.

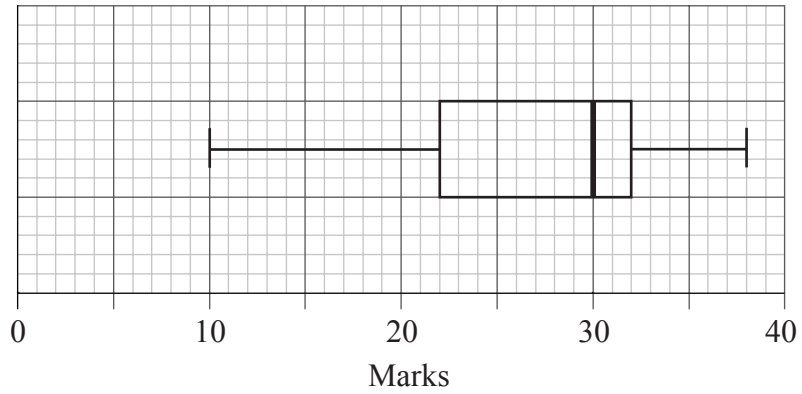
| Condition           | Line |
|---------------------|------|
| $m > 0$ and $c > 0$ |      |
| $m < 0$ and $c > 0$ |      |
| $m < 0$ and $c < 0$ |      |
| $m > 0$ and $c < 0$ |      |

[6 marks]

Working:



5. 56 students were given a test out of 40 marks. The teacher used the following box and whisker plot to represent the marks of the students.



- (a) Write down
- (i) the median mark;
  - (ii) the 75<sup>th</sup> percentile mark;
  - (iii) the range of marks. *[4 marks]*
- (b) Estimate the number of students who achieved a mark greater than 32. *[2 marks]*

*Working:*

*Answers:*

- (a) (i) \_\_\_\_\_  
(ii) \_\_\_\_\_  
(iii) \_\_\_\_\_
- (b) \_\_\_\_\_



6. A survey was carried out in a group of 200 people. They were asked whether they smoke or not. The collected information was organized in the following table.

|               | <b>Smoker</b> | <b>Non-smoker</b> |
|---------------|---------------|-------------------|
| <b>Male</b>   | 60            | 40                |
| <b>Female</b> | 30            | 70                |

One person from this group is chosen at random.

- (a) Write down the probability that this person is a smoker. *[2 marks]*
- (b) Write down the probability that this person is male given that they are a smoker. *[2 marks]*
- (c) Find the probability that this person is a smoker or is male. *[2 marks]*

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_



7. The weights of 90 students in a school were recorded. The information is displayed in the following table.

| Weight (kg)      | Number of students |
|------------------|--------------------|
| $40 \leq w < 50$ | 7                  |
| $50 \leq w < 60$ | 28                 |
| $60 \leq w < 70$ | 35                 |
| $70 \leq w < 80$ | 20                 |

- (a) Write down the mid interval value for the interval  $50 \leq w < 60$ . *[1 mark]*
  
- (b) Use your graphic display calculator to find an estimate for
  - (i) the mean weight;
  - (ii) the standard deviation. *[3 marks]*
  
- (c) Find the weight that is 3 standard deviations below the mean. *[2 marks]*

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) (i) \_\_\_\_\_  
(ii) \_\_\_\_\_
- (c) \_\_\_\_\_





8. The first term of an arithmetic sequence is 3 and the sum of the first two terms is 11.
- (a) Write down the second term of this sequence. *[1 mark]*
  - (b) Write down the common difference of this sequence. *[1 mark]*
  - (c) Write down the fourth term of this sequence. *[1 mark]*
  - (d) The  $n^{\text{th}}$  term is the first term in this sequence which is greater than 1000.  
Find the value of  $n$ . *[3 marks]*

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_
- (d) \_\_\_\_\_



9. Consider the universal set  $U = \{x \in \mathbb{N} \mid 3 < x < 13\}$ , and the subsets  $A = \{\text{multiples of } 3\}$  and  $B = \{4, 6, 12\}$ .

(a) List the elements of the following sets.

(i)  $A$

(ii)  $A \cap B'$

[2 marks]

(b) Write down one element of  $(A \cup B)'$ .

[2 marks]

(c) One of the statements in the table below is false. Indicate with an **X** which statement is false. Give a reason for your answer.

[2 marks]

|                      |  |
|----------------------|--|
| $n(A \cup B) = 4$    |  |
| $15 \in A'$          |  |
| $A \subset A \cup B$ |  |

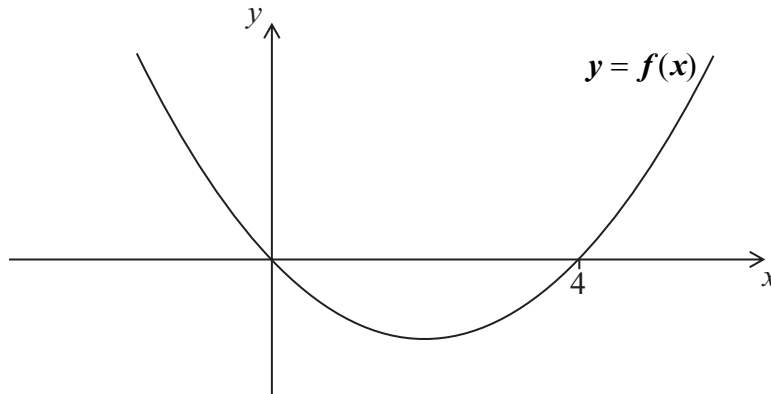
*Working:*

*Answers:*

- (a) (i) \_\_\_\_\_  
(ii) \_\_\_\_\_  
(b) \_\_\_\_\_  
(c) \_\_\_\_\_



10. The following is the graph of the quadratic function  $y = f(x)$ .



- (a) Write down the solutions to the equation  $f(x) = 0$ . [2 marks]
- (b) Write down the equation of the axis of symmetry of the graph of  $f(x)$ . [2 marks]
- (c) The equation  $f(x) = 12$  has two solutions. One of these solutions is  $x = 6$ . Use the symmetry of the graph to find the other solution. [1 mark]
- (d) The minimum value for  $y$  is  $-4$ . Write down the range of  $f(x)$ . [1 mark]

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_
- (d) \_\_\_\_\_



11. Consider the geometric sequence 16, 8,  $a$ , 2,  $b$ , ...

(a) Write down the common ratio. [1 mark]

(b) Write down the value of

(i)  $a$ ;

(ii)  $b$ . [2 marks]

(c) The sum of the first  $n$  terms is 31.9375. Find the value of  $n$ . [3 marks]

*Working:*

*Answers:*

(a) \_\_\_\_\_

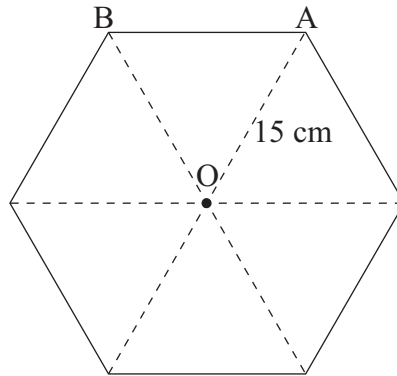
(b) (i) \_\_\_\_\_

(ii) \_\_\_\_\_

(c) \_\_\_\_\_



12. The base of a prism is a **regular hexagon**. The centre of the hexagon is O and the length of OA is 15 cm.



*diagram not to scale*

- (a) Write down the size of angle AOB. [1 mark]
- (b) Find the area of the triangle AOB. [3 marks]

The height of the prism is 20 cm.

- (c) Find the volume of the prism. [2 marks]

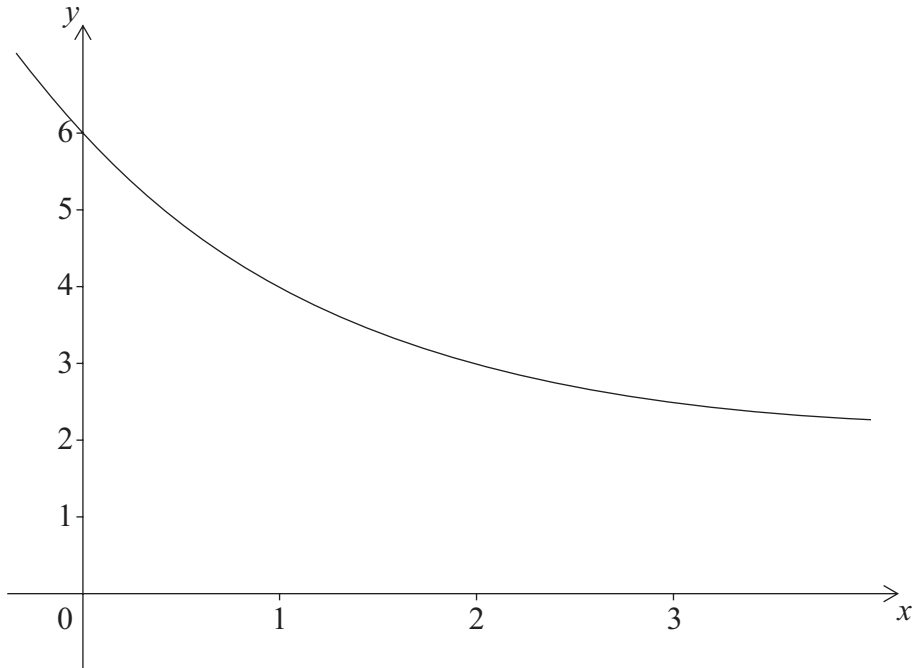
*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_



13. Consider the function  $f(x) = p(0.5)^x + q$  where  $p$  and  $q$  are constants. The graph of  $f(x)$  passes through the points  $(0, 6)$  and  $(1, 4)$  and is shown below.



- (a) Write down two equations relating  $p$  and  $q$ . [2 marks]
- (b) Find the value of  $p$  and of  $q$ . [2 marks]
- (c) Write down the equation of the horizontal asymptote to the graph of  $f(x)$ . [2 marks]

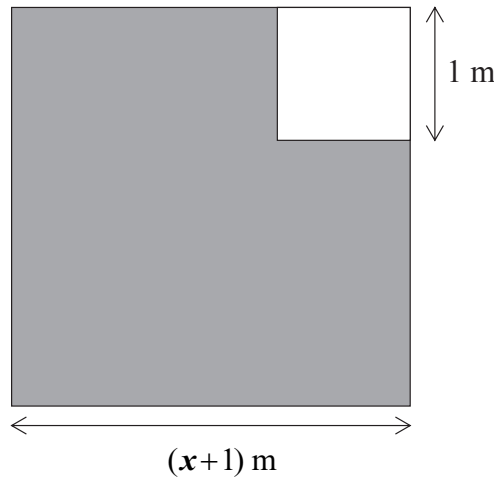
*Working:*

*Answers:*

- (a) \_\_\_\_\_
- \_\_\_\_\_
- (b) \_\_\_\_\_
- \_\_\_\_\_
- (c) \_\_\_\_\_
- \_\_\_\_\_



14. The length of a square garden is  $(x+1)$  m. In one of the corners a square of 1 m length is used only for grass. The rest of the garden is only for planting roses and is shaded in the diagram below.



*diagram not to scale*

The area of the shaded region is  $A$ .

- (a) Write down an expression for  $A$  in terms of  $x$ . [1 mark]
- (b) Find the value of  $x$  given that  $A = 109.25 \text{ m}^2$ . [3 marks]
- (c) The owner of the garden puts a fence around the shaded region. Find the length of this fence. [2 marks]

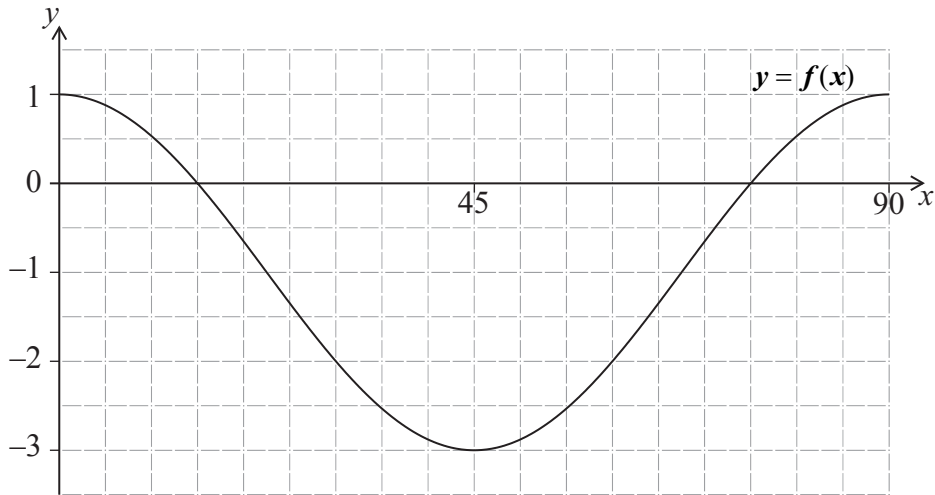
*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_



15. The graph of the function  $f(x) = 2\cos(4x) - 1$ , where  $0^\circ \leq x \leq 90^\circ$ , is shown on the diagram below.



(a) On the diagram draw the graph of the function  $g(x) = \sin(2x) - 2$ , for  $0^\circ \leq x \leq 90^\circ$ . [2 marks]

(b) Write down the number of solutions to the equation  $f(x) = g(x)$ , for  $0^\circ \leq x \leq 90^\circ$ . [1 mark]

(c) Write down one value of  $x$  for which  $f(x) > g(x)$ , for  $0^\circ \leq x \leq 90^\circ$ . [1 mark]

$f(x) < g(x)$  in the interval  $a < x < b$ .

(d) Use your graphic display calculator to find the value of

(i)  $a$ ;

(ii)  $b$ .

[2 marks]

Working:

Answers:

(b) \_\_\_\_\_

(c) \_\_\_\_\_

(d) (i) \_\_\_\_\_

(ii) \_\_\_\_\_

