Topic 7—Calculus 36 hrs

Aims

The aim of this section is to introduce students to the basic concepts and techniques of differential and integral calculus and their application.

Details

	Content	Amplifications/inclusions	Exclusions
7.1	Informal ideas of limit and convergence.	Only an informal treatment of limit and convergence, for example, 0.3, 0.33, 0.333, ... converges to $\frac{1}{3}$.	
	Definition of derivative as $f'(x)=\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$.	Use of this definition for derivatives of polynomial functions only. Other derivatives can be justified by graphical considerations using a GDC. Familiarity with both forms of notation, $\frac{dy}{dx}$ and $f'(x)$, for the first derivative.	
	Derivative of x^n $(n\in\mathbb{Q})$, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$.		
	Derivative interpreted as gradient function and as rate of change.	Finding equations of tangents and normals. Identifying increasing and decreasing functions.	

Topic 7—Calculus (continued)

	Content	Amplifications/inclusions	Exclusions
7.2	Differentiation of a sum and a real multiple of the functions in 7.1. The chain rule for composite functions. The product and quotient rules.		
	The second derivative.	Familiarity with both forms of notation, $\frac{d^2y}{dx^2}$ and $f''(x)$, for the second derivative.	
7.3	Local maximum and minimum points.	Testing for maximum or minimum using change of sign of the first derivative and using sign of the second derivative.	
	Use of the first and second derivative in optimization problems.	Examples of applications: profit, area, volume.	
7.4	Indefinite integration as anti-differentiation.		
	Indefinite integral of x^n $(n\in\mathbb{Q})$, $\sin x$, $\cos x$, $\frac{1}{x}$ and e^x.	$\int\frac{1}{x}dx=\ln x+C$, $x>0$.	
	The composites of any of these with the linear function $ax+b$.	Example: $f'(x)=\cos(2x+3)\Rightarrow f(x)=\frac{1}{2}\sin(2x+3)+C$.	

Topic 7—Calculus (continued)

	Content	Amplifications/inclusions	Exclusions
7.5	Anti-differentiation with a boundary condition to determine the constant term.	Example: if $\frac{dy}{dx} = 3x^2 + x$ and $y = 10$ when $x = 0$, then $y = x^3 + \frac{1}{2}x^2 + 10$.	
	Definite integrals.		
	Areas under curves (between the curve and the x-axis), areas between curves.	Only the form $\int_a^b y\,dx$.	$\int_a^b x\,dy$.
	Volumes of revolution.	Revolution about the x-axis only, $V = \int_a^b \pi y^2\,dx$.	Revolution about the y-axis; $V = \int_a^b \pi x^2\,dy$.
7.6	Kinematic problems involving displacement, s, velocity, v, and acceleration, a.	$v = \frac{ds}{dt}$, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$. Area under velocity–time graph represents distance.	
7.7	Graphical behaviour of functions: tangents and normals, behaviour for large $\|x\|$, horizontal and vertical asymptotes.	Both "global" and "local" behaviour.	Oblique asymptotes.
	The significance of the second derivative; distinction between maximum and minimum points.	Use of the terms "concave-up" for $f''(x) > 0$, "concave-down" for $f''(x) < 0$.	
	Points of inflexion with zero and non-zero gradients.	At a point of inflexion $f''(x) = 0$ and $f''(x)$ changes sign (concavity change). $f''(x) = 0$ is not a sufficient condition for a point of inflexion: for example, $y = x^4$ at $(0,0)$.	Points of inflexion where $f''(x)$ is not defined: for example, $y = x^{1/3}$ at $(0,0)$.