Topic 5—Vectors 16 hrs

Aims

The aim of this section is to provide an elementary introduction to vectors. This includes both algebraic and geometric approaches.

Details

	Content	Amplifications/inclusions	Exclusions
5.1	Vectors as displacements in the plane and in three dimensions.	Distance between points in three dimensions.	
	Components of a vector; column representation. $\boldsymbol{v}=\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}=v_1\boldsymbol{i}+v_2\boldsymbol{j}+v_3\boldsymbol{k}$.	Components are with respect to the unit vectors ***i***, ***j***, and ***k*** (standard basis).	
	Algebraic and geometric approaches to the following topics:		
	the sum and difference of two vectors; the zero vector, the vector $-\boldsymbol{v}$;	The difference of $\boldsymbol{v}$ and $\boldsymbol{w}$ is $\boldsymbol{v}-\boldsymbol{w}=\boldsymbol{v}+(-\boldsymbol{w})$.	
	multiplication by a scalar, $k\boldsymbol{v}$;		
	magnitude of a vector, $\lvert\boldsymbol{v}\rvert$;		
	unit vectors; base vectors ***i***, ***j***, and ***k***;		
	position vectors $\overrightarrow{\mathrm{OA}}=\boldsymbol{a}$.	$\overrightarrow{\mathrm{AB}}=\overrightarrow{\mathrm{OB}}-\overrightarrow{\mathrm{OA}}=\boldsymbol{b}-\boldsymbol{a}$.	

Topic 5—Vectors (continued)

	Content	Amplifications/inclusions	Exclusions
5.2	The scalar product of two vectors $\boldsymbol{v}\cdot\boldsymbol{w}=\lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert\cos\theta$; $\boldsymbol{v}\cdot\boldsymbol{w}=v_1w_1+v_2w_2+v_3w_3$.	The scalar product is also known as the "dot product" or "inner product".	Projections.
	Perpendicular vectors; parallel vectors.	For non-zero perpendicular vectors $\boldsymbol{v}\cdot\boldsymbol{w}=0$; for non-zero parallel vectors $\boldsymbol{v}\cdot\boldsymbol{w}=\pm\lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert$.	
	The angle between two vectors.		
5.3	Representation of a line as $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$.	Lines in the plane and in three-dimensional space. Examples of applications: interpretation of t as time and $\boldsymbol{b}$ as velocity, with $\lvert\boldsymbol{b}\rvert$ representing speed.	Cartesian form of the equation of a line: $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}$.
	The angle between two lines.		
5.4	Distinguishing between coincident and parallel lines.		
	Finding points where lines intersect.	Awareness that non-parallel lines may not intersect.	