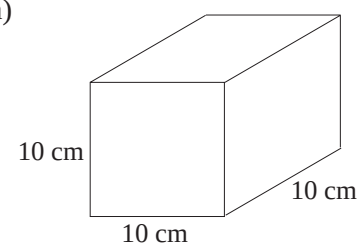
18 3-D Geometry

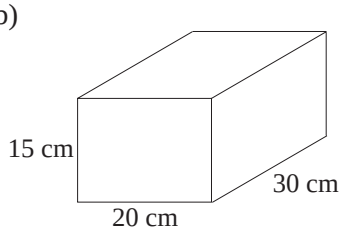
18.1 Using Pythagoras' Theorem and Trigonometry in Three Dimensions

1. Find the length of the longest rod that could be placed in each box shown below.

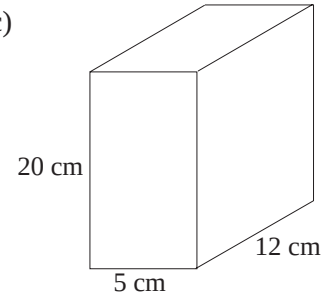
(a)



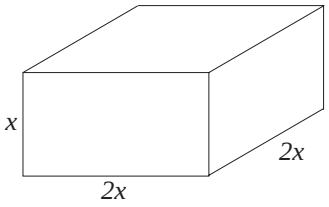
(b)



(c)

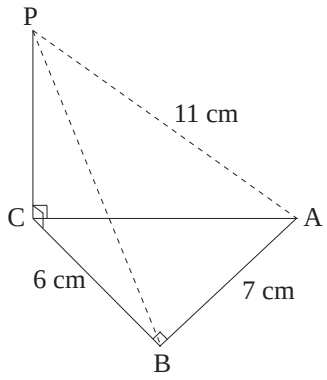


(d)



2. A square-based pyramid is made up of a square and four isosceles triangle with sides of lengths 6, 6 and 4 cm. Find the height of the pyramid.

3. The figure shows a triangle ABC, right-angled at B and lying in horizontal plane. P is a point vertically above C. Given that AB = 7 cm, BC = 6 cm and AP = 11 cm, calculate

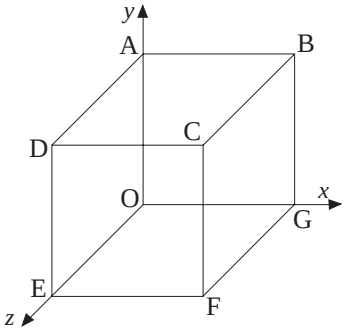


(a) the length of PC,

(b) $P\hat{A}C$,

(c) The angle of elevation of P from B.

4. This shape is a cube with OG = OE = OA = 2. O is the origin.



(a) Write down the three-dimensional coordinates of point F.

(b) Calculate the distance AC.

Not to Scale

(SEG)

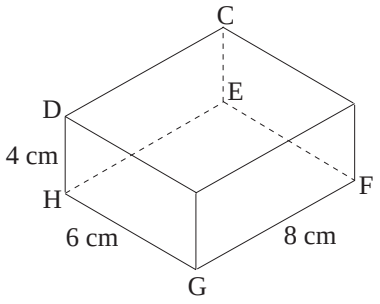
5. A rectangular box has a horizontal base EFGH. The corner D is vertically above H.

Given that DH = 4 cm, HG = 6 cm and GF = 8 cm, calculate

(a) $\hat{DGH}$,

(b) the length of HF,

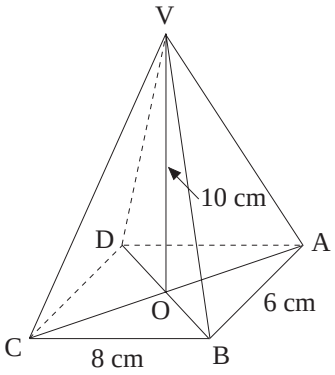
(c) $\hat{DFH}$.



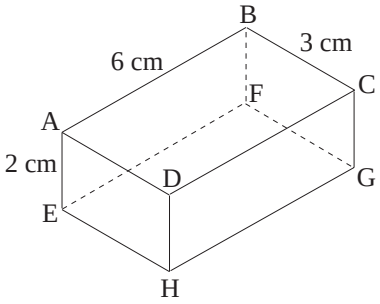
6. VABCD is a pyramid of height 10 cm. Its base is a rectangle with AB = 6 cm and BC = 8 cm. V is vertically above O, the point of intersection of the diagonals AC and BD. Find

(a) the length of VA,

(b) $\hat{VAO}$.



7. The diagram represents a rectangular box. Given that AB = 6 cm, BC = 3 cm and AE = 2 cm, calculate the length of the diagonal AG.



8. ABCDV is a right square-based pyramid.

M is the centre of the square base ABCD.

The (*x*, *y*, *z*) coordinates of A, B, and C are shown on the diagram.

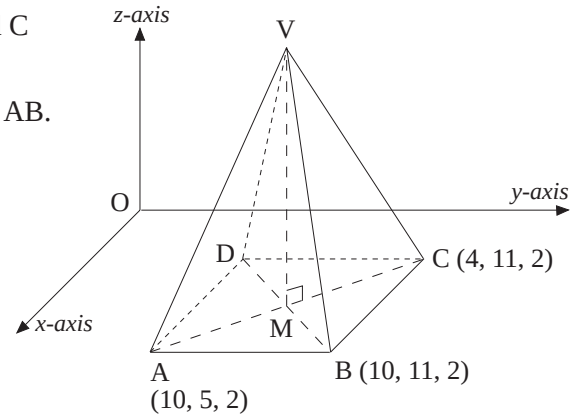
(a) (i) Write down the length of AB.

(ii) Write down the coordinates of D.

(b) Calculate the coordinates of M.

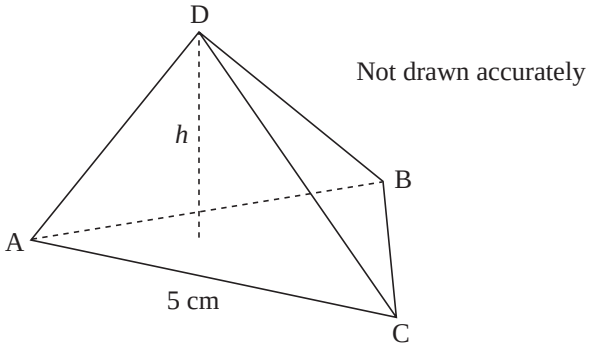
(c) The *z* coordinate of V is 9.

What is the height of the pyramid?



(NEAB)

9. ABCD is a triangular based pyramid. The base ABC is an equilateral triangle with side 5 cm. The volume of the pyramid is 36 cm^3.



Volume of a pyramid = $\frac{1}{3}$ × base area × perpemdicular height

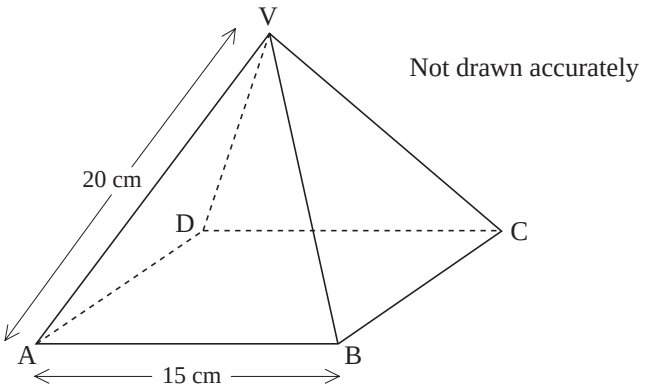
Calculate the perpendicular height, *h*, of the pyramid.

(AQA)

10. VABCD is a right pyramid on a square base. V is vertically above the centre of the square.

VA = VB = VC = VD = 20 cm

AB = 15 cm



Calculate the angle between the edge VA and the base ABCD.

(AQA)

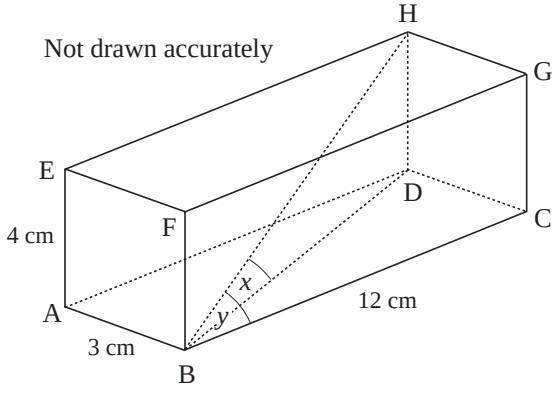
11. The diagram shows a cuboid.

AB = 3 cm, AE = 4 cm,
BC = 12 cm

(a) Find the length of BH.

(b) The angle between BH and BD is *x* and the angle between BH and BC is *y*.

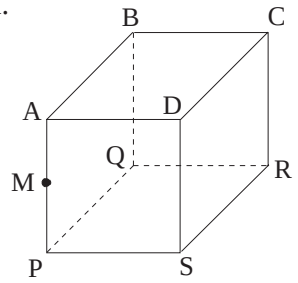
Which angle is bigger, *x* or *y*? You **must** show your working.



(AQA)

18.2 Angles and Planes

1. The cube shown in the figure has edges of length 20 cm. M is the mid-point of AP. Calculate



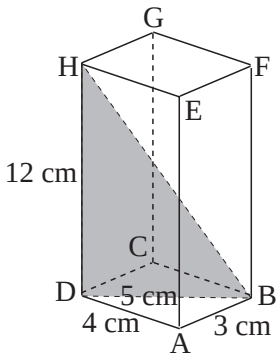
(a) the length of CM,

(b) the angle CMR,

(c) $M\hat{S}P$.

2. The diagram shows a rectangular box in which AB = 3 cm, AD = 4 cm, BD = 5 cm and DH = 12 cm.

Calculate the length of the straight line BH and



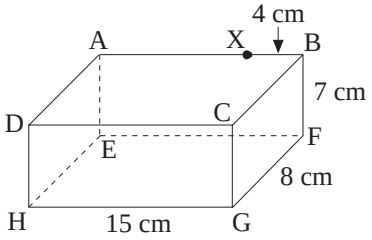
(a) $B\hat{D}C$

(b) $B\hat{H}C$

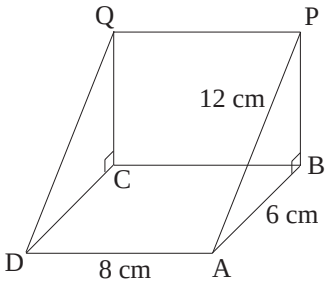
(c) $H\hat{B}D$

3. The diagram shows a rectangular box which has a horizontal base EFGH where HG = 15 cm, GF = 8 cm and BF = 7 cm.
X is a point on AB such that XB = 4 cm.

Calculate the angles CEG and GXF.



4. The diagram shows a right triangular prism with $A\hat{B}P = 90°$ and ABCD lying on a horizontal table. If AB = 6 cm, AD = 8 cm and AP = 12 cm, calculate

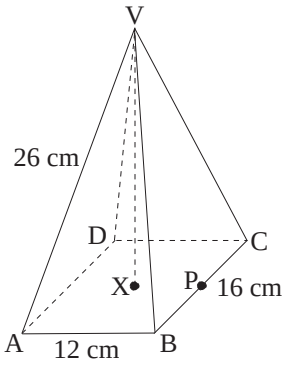


(a) $P\hat{A}B$,

(b) the length of PB,

(c) $P\hat{D}B$.

5. The diagram shows a right pyramid on a horizontal rectangular base ABCD.



Given that AB = 12 cm, BC = 16 cm and VA = 26 cm, calculate:

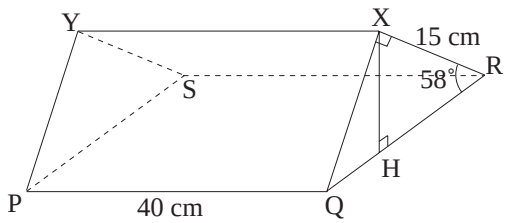
(a) the length of AX, where X is the mid-point of AC,

(b) the vertical height, VX, of the pyramid,

(c) the angle AVC,

(d) the length of VP where P is the mid-point of BC.

6.



The diagram shows a triangular prism.

The two triangular faces PSY and QRX are vertical. Two of the three rectangular faces PQXY and SRXY are at right angles, i.e. $R\hat{X}Q = 90°$, while the face PQRS is horizontal.

Given that the angle between the faces SRXY and PQRS, i.e. $X\hat{R}Q$, is $58°$, $X\hat{H}R = 90°$, RX = 15 cm and PQ = 40 cm, calculate

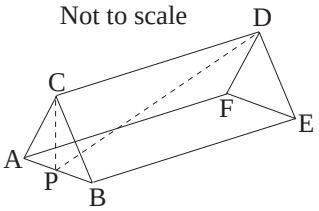
(a) QX,

(b) $X\hat{P}H$.

7. ABCDEF is a triangular prism, 10 cm long.

ABC is an equilateral triangle of side 3 cm.

P is the foot of the perpendicular from C to AB.



(a) Calculate the length of PD.

(b) Calculate the size of the angle between CE and PE.

(SEG)