International Baccalaureate
Baccalauréat International
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88127402

MATHEMATICAL STUDIES
STANDARD LEVEL

## PAPER 2

Wednesday 7 November 2012 (morning)
1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the Mathematical Studies SL information booklet is required for this paper.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is [ 90 marks].

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 13]

The table below shows the scores for 12 golfers for their first two rounds in a local golf tournament.

| Round 1 $(x)$ | 71 | 79 | 66 | 73 | 69 | 76 | 68 | 75 | 82 | 67 | 69 | 74 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Round 2 $(y)$ | 73 | 81 | 68 | 75 | 70 | 79 | 69 | 77 | 83 | 68 | 72 | 76 |

(a) (i) Write down the mean score in Round 1.
(ii) Write down the standard deviation in Round 1.
(iii) Find the number of these golfers that had a score of more than one standard deviation above the mean in Round 1.
(b) Write down the correlation coefficient, $r$.
(c) Write down the equation of the regression line of $y$ on $x$.

Another golfer scored 70 in Round 1.
(d) Calculate an estimate of his score in Round 2.

Another golfer scored 89 in Round 1.
(e) Determine whether you can use the equation of the regression line to estimate his score in Round 2. Give a reason for your answer.
2. [Maximum mark: 21]

In a college 450 students were surveyed with the following results

> 150 have a television
> 205 have a computer
> 220 have an iPhone
> 75 have an iPhone and a computer
> 60 have a television and a computer
> 70 have a television and an iPhone 40 have all three.
(a) Draw a Venn diagram to show this information. Use $T$ to represent the set of students who have a television, $C$ the set of students who have a computer and $I$ the set of students who have an iPhone.
(b) Write down the number of students that
(i) have a computer only;
(ii) have an iPhone and a computer but no television.
(c) Write down $n\left[T \cap(C \cup I)^{\prime}\right]$.
(d) Calculate the number of students who have none of the three.

Two students are chosen at random from the 450 students. Calculate the probability that
(e) (i) neither student has an iPhone;
(ii) only one of the students has an iPhone.

The students are asked to collect money for charity. In the first month, the students collect $x$ dollars and the students collect $y$ dollars in each subsequent month. In the first 6 months, they collect 7650 dollars. This can be represented by the equation $x+5 y=7650$.

In the first 10 months they collect 13050 dollars.
(f) (i) Write down a second equation in $x$ and $y$ to represent this information.
(ii) Write down the value of $x$ and of $y$.
(g) Calculate the number of months that it will take the students to collect 49500 dollars.
3. [Maximum mark: 19]

A contractor is building a house. He first marks out three points $\mathrm{A}, \mathrm{B}$ and C on the ground such that $\mathrm{AB}=5 \mathrm{~m}, \mathrm{AC}=7 \mathrm{~m}$ and angle $\mathrm{BAC}=112^{\circ}$.

diagram not to scale
(a) Find the length of BC.

He next marks a fourth point, $D$, on the ground at a distance of 6 m from B , such that angle BDC is $40^{\circ}$.

(b) Find the size of angle DBC.
(c) Find the area of the quadrilateral ABDC .

## (Question 3 continued)

The contractor digs up and removes the soil under the quadrilateral ABDC to a depth of 50 cm for the foundation of the house.
(d) Find the volume of the soil removed. Give your answer in $\mathbf{m}^{3}$.

To transport the soil removed, the contractor uses cylindrical drums with a diameter of 30 cm and a height of 40 cm .
(e) (i) Find the volume of a drum. Give your answer in $\mathbf{m}^{\mathbf{3}}$.
(ii) Find the minimum number of drums required to transport the soil removed. [5 marks]
4. [Maximum mark: 15]

A store recorded their sales of televisions during the 2010 football World Cup. They looked at the numbers of televisions bought by gender and the size of the television screens.
This information is shown in the table below; $S$ represents the size of the television screen in inches.

|  | $S \leq 22$ | $22<S \leq 32$ | $32<S \leq 46$ | $S>46$ | Total |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Female | 65 | 100 | 40 | 15 | 220 |
| Male | 20 | 65 | 140 | 55 | 280 |
| Total | 85 | 165 | 180 | 70 | 500 |

The store wants to use this information to predict the probability of selling these sizes of televisions for the 2014 football World Cup.
(a) Use the table to find the probability that
(i) a television will be bought by a female;
(ii) a television with a screen size of $32<S \leq 46$ will be bought;
(iii) a television with a screen size of $32<S \leq 46$ will be bought by a female;
(iv) a television with a screen size greater than 46 inches will be bought, given that it is bought by a male.
[6 marks]

The manager of the store wants to determine whether the screen size is independent of gender. A Chi-squared test is performed at the $1 \%$ significance level.
(b) Write down the null hypothesis.
(c) Show that the expected frequency for females who bought a screen size of $32<S \leq 46$, is 79 , correct to the nearest integer.
(d) Write down the number of degrees of freedom.
(e) Write down the $\chi^{2}$ calculated value.
(f) Write down the critical value for this test.
(g) Determine if the null hypothesis should be accepted. Give a reason for your answer.
5. [Maximum mark: 22]

Consider the function $g(x)=b x-3+\frac{1}{x^{2}}, x \neq 0$.
(a) Write down the equation of the vertical asymptote of the graph of $y=g(x)$.
(b) Write down $g^{\prime}(x)$.

The line $T$ is the tangent to the graph of $y=g(x)$ at the point where $x=1$. The gradient of $T$ is 3 .
(c) Show that $b=5$.
(d) Find the equation of $T$.
(e) Using your graphic display calculator find the coordinates of the point where the graph of $y=g(x)$ intersects the $x$-axis.
(f) (i) Sketch the graph of $y=g(x)$ for $-2 \leq x \leq 5$ and $-15 \leq y \leq 25$, indicating clearly your answer to part (e).
(ii) Draw the line $T$ on your sketch.
(g) Using your graphic display calculator find the coordinates of the local minimum point of $y=g(x)$.
(h) Write down the interval for which $g(x)$ is increasing in the domain $0<x<5$.

