## MARKSCHEME

## May 2012

## MATHEMATICAL STUDIES

## Standard Level

## Paper 2

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## Paper 2 Markscheme

Instructions to Examiners

Notes: If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

## 1 Abbreviations

M Marks awarded for Method
A Marks awarded for an Answer or for Accuracy
$\boldsymbol{R} \quad$ Marks awarded for clear Reasoning
G Marks awarded for correct solutions obtained from a Graphic Display Calculator, irrespective of working shown.

AG Answer Given in the question and consequently, marks not awarded.
ft Marks that can be awarded as follow through from previous results in the question.
2 Method of Marking
(a) All marking must be done in scoris using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
(b) If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the $\boldsymbol{A} \boldsymbol{0}$ annotation, otherwise full annotations must be shown.
(c) Working crossed out by the candidate should not be awarded any marks.
(d) Where candidates have written two solutions to a question, only the first solution should be marked.
(e) If correct working results in a correct answer but then further working is developed, full marks may not always be awarded. Full marks will be awarded if the candidate shows correct working leading to the correct answer. See also section 4(c).

Example: Calculate the gradient of the line passing through the points $(5,3)$ and $(0,9)$.

| Markscheme | Candidates' Scripts ${ }^{\text {a }}$ Marking |
| :---: | :---: |
| $\begin{equation*} \frac{9-3}{0-5} \tag{M1} \end{equation*}$ <br> Award (M1) for correct substitution in gradient formula $\begin{equation*} =-\frac{6}{5} \tag{A1} \end{equation*}$ | (i) $\frac{9-3}{0-5}=-\frac{6}{5}$ <br> (MI) <br> Gradient is $=-\frac{6}{5}$ <br> (There is clear understanding of the gradient.) $y=-\frac{6}{5} x+9$ <br> (ii) $\begin{align*} & \frac{9-3}{0-5}=-\frac{6}{5}  \tag{M1}\\ & y=-\frac{6}{5} x+9 \tag{A0} \end{align*}$ |

## 3 <br> Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, follow through (ft) marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with '(ft)'.
(a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
(b) If an answer resulting from follow through is extremely unrealistic (e.g. negative distances or incorrect by large order of magnitude) then the final $\boldsymbol{A}$ mark should not be awarded.
(c) If a question is transformed by an error into a different, much simpler question then follow through may not apply.
(d) To award follow through marks for a question part, there must be working present for that part. An isolated follow through answer, without working is regarded as incorrect and receives no marks even if it is approximately correct.
(e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. The markscheme will clearly indicate where this applies.
(f) Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

| Markscheme |  | Candida | tes' Scripts | Marking |
| :---: | :---: | :---: | :---: | :---: |
| (a) $\frac{\sin A}{3}=\frac{\sin 30}{4} \quad(\boldsymbol{M 1})($ A1 $)$ |  | $\underline{\sin A}=\frac{\sin 30}{3}$ |  | $(M 1)(A 0)$ |
| Award (M1) for substitution in sine rule formula, (A1) for correct substitutions. |  |  |  | (use of sine rule but with wrong values) |
| $A=22.0^{\circ}(22.0243 \ldots)($ A1) (G2) |  | $A=41.8^{\circ}$ | (A0) <br> (Note: the $2^{\text {nd }}(\mathbf{A 1})$ here was not marked (ft) and cannot be awarded because there was an earlier error in the same question part.) |  |
| (b) $\quad x=7 \tan \left(22.0243 \ldots .^{\circ}\right.$ ) (M1) | (b) | case (i) | $x=7 \tan 41.8^{\circ}$ | (M1) |
| $=2.83$ (2.83163 ...) (A1)(ft) |  | $=6.26$ |  | $\begin{aligned} & (A 1)(f t) \\ & (G 0) \end{aligned}$ |
|  |  |  | 6.26 | since no working shown |

## 4 Using the Markscheme

(a) $\boldsymbol{A}$ marks are dependent on the preceding $\boldsymbol{M}$ mark being awarded, it is not possible to award (M0)(A1). Once an (M0) has been awarded, all subsequent $\boldsymbol{A}$ marks are lost in that part of the question, even if calculations are performed correctly, until the next $\boldsymbol{M}$ mark.
The only exception to this will be for an answer where the accuracy is specified in the question - see section 5 .
(b) $\boldsymbol{A}$ marks are dependent on the $\boldsymbol{R}$ mark being awarded, it is not possible to award $(\boldsymbol{A} \boldsymbol{1})(\boldsymbol{R O})$. Hence the (A1) cannot be awarded for an answer which is correct when no reason or the wrong reason is given.
(c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will not always receive full marks, these unsupported answers are designated $\boldsymbol{G}$ in the mark scheme as an alternative to the full marks. Example (M1)(A1)(A1)(G2).

Example: Using trigonometry to calculate an angle in a triangle.

| Markscheme | Candidates' Scripts ${ }^{\text {a }}$ Marking |
| :---: | :---: |
| (a) $\frac{\sin A}{3}=\frac{\sin 30}{4} \quad($ M1 $)($ A1 $)$ <br> Award (M1) for substitution in sine rule formula, (A1) for correct substitutions. $A=22.0^{\circ}(22.0243 \ldots)(\boldsymbol{A 1})(\boldsymbol{G} 2)$ | (i) $\begin{align*} & \frac{\sin A}{3}=\frac{\sin 30}{4} \\ & A=22.0^{\circ} \tag{A1} \end{align*}$ <br> (M1)(A1) <br> (ii) $A=22.0^{\circ}$ <br> (G2) <br> Note: $\boldsymbol{G}$ marks are used only if no working has been shown and the answer is correct. |

(d) Alternative methods may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.
Where alternative methods for complete questions are included in the markscheme, they are indicated by 'OR' etc.
(e) Unless the question specifies otherwise, accept equivalent forms. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$. On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:
the 3 significant figure answer worked through from full calculator display;
the exact value (for example $\sqrt{3}$ if applicable);
the full calculator display in the form $2.83163 \ldots$ as in the example above.
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a different 3 significant figure answer, these solutions will also be given.
(f) As this is an international examination, all valid alternative forms of notation should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; 1•7;1,7.

Different descriptions of an interval: $3<x<5$; $(3,5) ;$ ] 3,5 [.
Different forms of notation for set properties (e.g. complement): $A^{\prime} ; \bar{A} ; A^{c} ; U-A ;(A ; U \backslash A$.
Different forms of logic notation: $\neg p ; p^{\prime} ; \widetilde{p} ; \bar{p} ; \sim p$.

$$
p \Rightarrow q ; p \rightarrow q ; q \Leftarrow p .
$$

(g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt and exception should be raised through scoris to the team leader.

As from Nov 11 there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

## 5

## Accuracy of Answers

Incorrect accuracy should be penalized once only in each question according to the rules below.
Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the candidate's unrounded answer is seen and would round to the required 3 sf answer, then award (A1) and ignore subsequent rounding.
Note: The unrounded answer may appear in either the working box or on the final answer line.
2. If the candidate's unrounded answer is not seen then award (A1) if the answer given is correctly rounded to 2 or more significant figures, otherwise (A0).
Note: If the candidate's unrounded answer is not seen and is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.
3. If a correct 2 sf answer is used in subsequent parts, then working must be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarised in the table below and illustrated in the examples following.

|  | If candidates final answer is given... |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Exact or correct to 3 or more sf | Incorrect to 3sf | $\begin{aligned} & \text { Correct to } \\ & 2 \mathrm{sf}^{3} \end{aligned}$ | Incorrect to 2sf | Correct or incorrect to 1sf |
| Unrounded answer seen ${ }^{1}$ | Award the final (A1) irrespective of correct or incorrect rounding |  |  |  |  |
| Unrounded answer not seen ${ }^{2}$ | (A1) | (AO) | (A1) | (A0) | (A0) |
| Treatment of subsequent parts | As per MS | Treat as follow through, only if working is seen ${ }^{3}$ |  |  |  |

## Examples:




Example: $A B C$ is a right angled triangle with angle $A B C=90^{\circ}, A C=32 \mathrm{~cm}$ and $A B=30 \mathrm{~cm}$. Find (a) the length of BC , (b) The area of triangle ABC.


Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.
e.g. chi-squared, correlation coefficient, mean

| Markscheme | Candidates' Scripts |  | Marking |
| :--- | :--- | :--- | :--- |
| Chi squared | (a) 7.7 | (G2) |  |
| 7.68 (7.67543...) (A2) | (b) 7.67 | (G1) |  |
|  | (c) 7.6 | (G1) |  |
|  | (d) 8 | (G0) |  |
|  | (e) 7 | (G0) |  |
|  | (e) 7.66 | (G0) |  |

Regression line

| Markscheme |  | Candidates' Scripts | Marking |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & y=0.888 x+13.5 \quad \text { A2) } \\ & (y=0.887686 \ldots x+13.4895 \ldots) \end{aligned}$ <br> If an answer is not in the form of an equation award at most (A1)(A0). | (a) | $y=0.89 x+13$ | (G2) <br> (both accepted) |
|  | (b) | $y=0.88 x+13$ | (G1) <br> (one rounding error) |
|  | (c) | $y=0.88 x+14$ | (G1) <br> (rounding error repeated) |
|  | (d) | (i) $y=0.9 x+13$ |  |
|  |  | (ii) $y=0.8 x+13$ | (G1) <br> (1sf not accepted) |
|  | (e) | $0.88 x+13$ | (GO) <br> error and not an equation) |

Maximum/minimum/points of intersection


Rounding of an exact answer to 3 significant figures should be accepted if performed correctly. Exact answers such as $\frac{1}{4}$ can be written as decimals to fewer than three significant figures if the result is still exact. Reduction of a fraction to its lowest terms is not essential, however where an answer simplifies to an integer this is expected.

Ratios of $\pi$ and answers taking the form of square roots of integers or any rational power of an integer (e.g. $\sqrt{13}, 2^{2 / 3}, \sqrt[4]{5}$,) may be accepted as exact answers. All other powers (e.g. of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a (ft) mark regardless of an immediately preceding (MO).

## 6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final $\boldsymbol{A}$ mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2dp.

| Markscheme | Candidates' Scripts |  | Marking |
| :---: | :---: | :---: | :---: | :---: |
| $\$ 231.62(231.6189)$ | (A1) | (i) 231.6 | (A0) |
|  |  | (ii) 232 | (A0) |
|  |  | (Correct rounding to incorrect level) |  |
|  | (iii) 231.61 | (A0) |  |
|  | (iv) 232.00 |  |  |
|  |  |  | (Parts (iii) and (iv) are both incorrect rounding to correct level) |

## 7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final $\boldsymbol{A}$ mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for lack of units or incorrect units.
The units are considered only when the numerical answer is awarded (A1) under the accuracy rules given in Section 5.

## Example:

| Markscheme |  | Candidates' Scripts |  |  |  | Marking |
| :--- | :---: | :--- | :---: | :---: | :---: | :---: |
| (a) $37000 \mathrm{~m}^{2}$ | (A1) | (a) $36000 \mathrm{~m}^{2}$ | (Ancorrect answer so units not considered) |  |  |  |
| (b) $3200 \mathrm{~m}^{3}$ | (A1) | (b) $3200 \mathrm{~m}^{2}$ | (A0) |  |  |  |
| (Incorrect units) |  |  |  |  |  |  |

## If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.

## 8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

## QUESTION 1

(a)

(A1) for three circles and a rectangle ( $U$ need not be seen)
(A1) for 5
(A1) for 3,8 and 12
(A1) for 16, 26 and 29 OR 32, 46, 54 placed outside the circles.
(A4)
[4 marks]
Note: Accept answers given as decimals or fractions.
(b) $100-(16+26+29)-(8+5+3+12)$
(M1)
$100-71-28$
Note: Award (M1) for correct expression. Accept equivalent expressions, for example $100-71-28$ or $100-(71+28)$.
$=1$
(A1)(ft)(G2)
[2 marks]
Note: Follow through from their Venn diagram but only if working is seen.
(c) $16+26+29$
(M1)
Note: Award (M1) for 16, 26, 29 seen.

$$
=71
$$

Note: Follow through from their Venn diagram but only if working is seen.
(d) $16+3+26$

Note: Award (M1) for their 16, 3, 26 seen.

$$
=45
$$

(A1)(ft)(G2)
[2 marks]
Note: Follow through from their Venn diagram but only if working is seen.

Question 1 continued
(e) True
(A1)(ft)
(R1)(ft)

## OR

$46+54-17=83$
(R1)(ft) [2 marks]
Note: Do not award (A1)(R0). Follow through from their Venn diagram.
(f) $28 \%$ of 120000

$$
\begin{equation*}
=33600 \tag{A1}
\end{equation*}
$$

$\%$ error $=\frac{(34000-33600)}{33600} \times 100$
(M1)

Note: Award (M1) for 28 seen (may be implied by 33600 seen), award (M1) for correct substitution of their 33600 in the percentage error formula. If an error is made in calculating 33600 award a maximum of $(\mathbf{M 1})(\boldsymbol{A 0})(\mathbf{M 1})(\boldsymbol{A 0})$, the final accuracy mark is lost.

## OR

$\frac{34000}{120000} \times 100$
(M1)
$=28.3(28.3333 \ldots)$
$\%$ error $=\frac{(28.3333 \ldots-28)}{28} \times 100$
$=1.19 \%(1.19047 \ldots)$
Note: \% sign not required. Accept 1.07 (1.0714...) with use of 28.3 . 1.18 with use of 28.33 and 1.19 with use of 28.333 .

Award (G3) for 1.07, 1.18 or 1.19 seen without working.

## QUESTION 2

(a) $\mathrm{H}_{0}$ : Gender and choice of fafterschool sport are independent.
(A1)
Note: Accept "not associated", do not accept "not related", "not correlated", or "not linked". Accept "the relation between gender and sport is independent".
(b) $\frac{85}{120} \times \frac{48}{120} \times 120\left(\frac{85 \times 48}{120}\right)$
(M1)

Note: Award (M1) for correct expression.

$$
=34
$$

(A1)(G2)
(c) 2
(A1)(ft)
[1 mark]
(d) 5.99 (5.991)
(e) $2.42(2.42094 \ldots)$
(R1)(A1)(ft)
(f) Since $2.42<5.99$ therefore accept (do not reject) $\mathrm{H}_{0}$

Note: The numerical values need not be seen, but must be consistent with their parts (d) and (e).

## OR

p-value $0.298>0.05$ therefore accept (do not reject) $\mathrm{H}_{0}$
(R1)(A1)
[2 marks]
Note: p-value comparison may not be used as part of a follow through solution.
Do not award (A1)(R0). Follow through from parts (c), (d) and (e).

## Question 2 continued

(g) (i) $\frac{35}{120}\left(\frac{7}{24}, 0.292,29.2 \%\right)(0.291666 \ldots)$
(A1)
(ii) $\frac{25}{120}\left(\frac{5}{24}, 0.208,20.8 \%\right)(0.208333 \ldots)$
(A1) [2 marks]
(h) (i) $\frac{48}{120} \times \frac{47}{119}$
(A1)(M1)

Note: Award (A1) for two correct fractions, (M1) for multiplying their two fractions.

$$
=\frac{94}{595}(0.158,15.8 \%)(0.157983 \ldots)
$$

(A1)(G2)
(ii) $\frac{73}{120} \times \frac{72}{119}$
(M1)

Note: Award (M1) for multiplying correct fractions.
If sampling with replacement has been used in both parts (h)(i) and (h)(ii) do not penalise in part (h)(ii). Award a maximum of (MI)(A1)(ft).

$$
=\frac{219}{595}(0.368,36.8 \%)(0.368067 \ldots)
$$

QUESTION 3 Units are required in parts (a), (c) and (g).

$$
\text { (a) } \begin{aligned}
& \pi \times 4^{2} \\
& =50.3(16 \pi) \mathrm{cm}^{2}(50.2654 \ldots)
\end{aligned}
$$

Note: Award (M1) for correct substitution in area formula. The answer is $50.3 \mathrm{~cm}^{2}$, the units are required.
(b) $50.265 \ldots \times 8$

Note: Award (M1) for correct substitution in the volume formula.

$$
\begin{aligned}
& =402.123 \ldots \\
& =402\left(\mathrm{~cm}^{3}\right)
\end{aligned}
$$

Note: Both the unrounded and the rounded answer must be seen for the (A1) to be awarded. The units are not required
(c) $2 \times \pi \times 4 \times 8+2 \times \pi \times 4^{2}$

Note: Award (M1) for correct substitution in the curved surface area formula, (M1) for adding the area of their two bases.
$=302 \mathrm{~cm}^{2}\left(96 \pi \mathrm{~cm}^{2}\right)(301.592 \ldots)$
Notes: The answer is $302 \mathrm{~cm}^{2}$, the units are required. Do not penalise for missing or incorrect units if penalised in part (a). Follow through from their answer to part (a).
(d) $\frac{1}{3} \pi \times 6^{2} \times \mathrm{OC}=402$

Note: Award (M1) for correctly substituted volume formula, (M1) for equating to 402 (402.123...).

$$
\mathrm{OC}=10.7(\mathrm{~cm})\left(10 \frac{2}{3}, 10.6666 \ldots\right)
$$

(M1)
(A1)(G2)
[2 marks]
(M1)
(A1)
(AG)
[2 marks]
(M1)(M1)
(A1)(ft)(G2)
(M1)(M1)
(A1)(G2)
[3 marks]

## Question 3 continued

(e) $\tan \mathrm{BCO}=\frac{6}{10.66 \ldots}$
(M1)

Note: Award (M1) for use of correct tangent ratio.

$$
\mathrm{B} \hat{C} \mathrm{O}=29.4^{\circ}(29.3577 \ldots)
$$

Notes: Accept $29.3^{\circ}(29.2814 \ldots)$ if 10.7 is used.
$\square$
Note: An acceptable alternative method is to calculate CB first and then angle BCO. Allow follow through from parts (d) and (f). Answers range from $29.2^{\circ}$ to $29.5^{\circ}$.
(f) $\mathrm{CB}=\sqrt{6^{2}+(10.66 \ldots)^{2}}$
(M1)
OR
$\sin 29.35 \ldots=\frac{6}{C B}$
(M1)

OR
$\cos 29.35 \ldots \circ=\frac{10.66 \ldots}{C B}$
$\mathrm{CB}=12.2(\mathrm{~cm})(12.2383 . .$.
(A1)(ft)(G2)
Note: Accept 12.3 (12.2674...) if 10.7 (and/or 29.3) used.
Follow through from part (d) or part (e) as appropriate.
(g) $\pi \times 6 \times 12.2383 \ldots+\pi \times 6^{2}$
(M1)(M1)(M1)
Note: Award (M1) for correct substitution in curved surface area formula,
(M1) for correct substitution in area of circle formula,
(M1) for addition of the two areas.
$=344 \mathrm{~cm}^{2}$ (343.785 ...)
(A1)(ft)(G3)
[4 marks]
Note: The answer is $344 \mathrm{~cm}^{2}$, the units are required. Do not penalise for missing or incorrect units if already penalised in either part (a) or (c).
Accept $345 \mathrm{~cm}^{2}$ if 12.3 is used and $343 \mathrm{~cm}^{2}$ if 12.2 is used. Follow through from their part (f).

## QUESTION 4

## Part A

$\square$
(a) (i) $u_{1}+5 d=100$
(ii) $u_{1}+9 d=124$
(A1)
[2 marks]
$\square$
(b) (i) 6
(G1)(ft)
(ii) 70
(G1)(ft)
[2 marks]
Notes: Follow through from their equations in parts (a) and (b) even if working not seen. Their answers must be integers. Award (M1)(A0) for an attempt to solve two equations analytically.
(c) $S_{20}=\frac{20}{2}(2 \times 70+(20-1) \times 6)$
(M1)(A1)(ft)
Note: Award (M1) for substituted sum of AP formula, (A1)(ft) for their correct substituted values.

$$
=2540
$$

Note: Follow through from their part (b).
(d) $\frac{n}{2}(2 \times 70+(n-1) \times 4)=1600$

Note: Award (M1) for substituted sum of AP formula, (A1) for their correct substituted values.
$4 n^{2}+136 n-3200=0$
(M1)
Note: Award (M1) for this equation (or other equivalent expanded quadratic) seen, may be implied if correct final answer seen.

$$
\begin{equation*}
n=16 \tag{A1}
\end{equation*}
$$

Note: Do not award the final (A1) for $n=16,-50$ given as final answer, award (G2) if $n=16,-50$ given as final answer without working.

## Question 4 continued

## Part B

(a) 9,27
(b) 3
(A1)
[1 mark]
(c) $1 \times 3^{6}$
(M1)
(A1)(ft)(G2)
[2 marks]

Note: Award (M1) for correctly substituted GP formula.
Follow through from their answer to part (b).
(d) $\frac{1\left(3^{7}-1\right)}{(3-1)}$
(M1)

Note: Award (M1) for correctly substituted GP formula. Accept sum $1+3+9+27+\ldots+729$.
If lists are used, award (M1) for correct list that includes 1093.
(1, 4, 13, 40, 121, 364, 1093, 3280...)
$=1093$
(A1)(ft)(G2)
[2 marks]
Note: Follow through from their answer to part (b).
Note: For consistent use of $n=6$ from part (c) (243) to part (d) leading to an answer of 364 , treat as double penalty and award $(M 1)(A 1)(\mathbf{f t})$ if working is shown.
(e) $\frac{1\left(3^{n}-1\right)}{(3-1)}=29524$

Note: Award (M1) for correctly substituted GP formula.
If lists are used, award (M1) for correct list that includes 29524.
$(1,4,13,40,121,364,1093,3280,9841,29524,88573 \ldots)$.
Accept alternative methods, for example continuation of sum in part (d).

$$
\begin{equation*}
n=10 \tag{A1}
\end{equation*}
$$

Note: Follow through from their answer to part (b).
Exact time $=12: 45$
(A1)(ft)(G2)

## QUESTION 5

(a)

(A1) for indication of window_and labels. (A1) for smooth curve that does not enter the first quadrant, the curve must consist of one line only.
(A1) for $x$ and $y$ intercepts in approximately correct positions (allow $\pm 0.5$ ).
(A1) for local maximum and minimum in approximately correct position.
(minimum should be $0 \leq x \leq 1$ and $-2 \leq y \leq-4$ ), the $y$-coordinate of the maximum should be $0 \pm 0.5$.
(A4) [4 marks]
continued...

## Question 5 continued

(b) $-\frac{1}{3}(-1)^{3}+\frac{5}{3}(-1)^{2}-(-1)-3$
(M1)

Note: Award (M1) for substitution of -1 into $f(x)$

$$
=0
$$

(A1)(G2)
[2 marks]
(c) $(0,-3)$
(A1)

## OR

$$
\begin{equation*}
x=0, y=-3 \tag{A1}
\end{equation*}
$$

[1 mark]
Note: Award (A0) if brackets are omitted.
(d) $\quad f^{\prime}(x)=-x^{2}+\frac{10}{3} x-1$
(A1)(A1)(A1) [3 marks]
Note: Award (A1) for each correct term. Award (A1)(A1)(A0) at most if there are extra terms.
(e) $\quad f^{\prime}(-1)=-(-1)^{2}+\frac{10}{3}(-1)-1$
$=-\frac{16}{3}$
Note: Award (M1) for substitution of $x=-1$ into correct derivative only. The final answer must be seen.
(f) $\quad f^{\prime}(-1)$ gives the gradient of the tangent to the curve at the point with $x=-1 . \quad(\boldsymbol{A 1})(\boldsymbol{A 1})$

Note: Award (A1) for "gradient (of curve)", (A1) for "at the point with $x=-1$ ". Accept "the instantaneous rate of change of $y$ " or "the (first) derivative".

Question 5 continued
(g) $y=-\frac{16}{3} x+c$
(M1)

Note: Award (M1) for $-\frac{16}{3}$ substituted in equation.

$$
\begin{aligned}
& 0=-\frac{16}{3} \times(-1)+c \\
& c=-\frac{16}{3} \\
& y=-\frac{16}{3} x-\frac{16}{3}
\end{aligned}
$$

Note: Accept $y=-5.33 x-5.33$.
OR

$$
(y-0)=\frac{-16}{3}(x+1)
$$

(M1)(A1)(G2)

Note: Award (M1) for $-\frac{16}{3}$ substituted in equation, (A1) for correct equation. Follow through from their answer to part (b).
Accept $y=-5.33(x+1)$.
Note: Accept equivalent equations.
(h) (A1)(ft) for a tangent to their curve drawn.
(A1)(ft) for their tangent drawn at the point $x=-1$.
(A1)(ft)(A1)(ft)
[2 marks]
Note: Follow through from their graph. The tangent must be a straight line otherwise award at $\operatorname{most}(\boldsymbol{A O})(\boldsymbol{A 1})$.
(i) (i) $\quad a=\frac{1}{3}$
(ii) $b=3$
(G1) [2 marks]
Note: If $a$ and $b$ are reversed award (A0)(A1).
(j) $\quad f(x)$ is increasing
(A1) [1 mark]

