

MARKSCHEME

November 2011

MATHEMATICAL STUDIES

Standard Level

Paper 1

Deadline for all marking to be completed is 06/12/11.

Ignore deadline on IBIS

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Paper 1 Markscheme Instructions to Examiners

Notes:

If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

The number of marks for each question is 6.

1 Abbreviations

The markscheme may make use of the following abbreviations:

- **M** Marks awarded for **Method**
- A Marks awarded for an **Answer** or for **Accuracy**
- C Marks awarded for Correct answers (irrespective of working shown)
- **R** Marks awarded for clear **Reasoning**
- ft Marks that can be awarded as **follow through** from previous results in the question

2 Method of Marking

- (a) All marking must be done in scoris using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- (b) If the candidate has full marks on a question use the C6 annotation, if the candidate has made an attempt but scores zero marks use C0. If there is no attempt use the No response button. If a candidate does not score full or zero marks then full annotations **MUST** be shown.
- (c) In this paper, if the **correct answer is seen on the answer line the** maximum mark is awarded. **There is no need to check the working!** Award *C* marks and move on.
- (d) If the answer does not appear on the answer line, but the correct answer is seen in the working box with no subsequent working, award the maximum mark.
- (e) If the **answer is wrong**, marks should be awarded for the working according to the markscheme.
- (f) Working crossed out by the candidate should not be awarded any marks. Where candidates have written two solutions to a question, only the first solution should be marked.
- (g) A correct answer in the working box transcribed inaccurately to the answer line can receive full marks.
- (h) If correct working results in a correct answer **in the working box** but then further working is developed, full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost, however, a statement on the answer line should always be taken as the candidate's final decision on the answer **as long as it is unambiguous**.

Accuracy of numerical answers is an exception to this rule- see Section 5.

Example: Factorise $x^2 - 5x - 6$

Marksc	heme		Candidates' Scripts	Marking
(x-6)(x+1)	(A1)(A1)	(i)	Answer line: $(x+6)(x+1)$	$(A\theta)(A1)$
		(ii) Working box: $(x-6)(x+1)$		(A1)
			followed by $x = 6$ and -1 , or just $6, -1$ in either working box or on answer line.	<i>(A0)</i>

3 Follow through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through** (**ft**) marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with '(**ft**)'.

- (a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- (b) If an answer resulting from follow through is extremely unrealistic (*e.g.* negative distances or incorrect by large order of magnitude) then the final *A* mark should not be awarded.
- (c) If a question is transformed by an error into a **different**, **much simpler question** then follow through may not apply.
- (d) To award follow through marks for a question part, there must be working present for that part. An isolated follow through answer, without working is regarded as incorrect and receives no marks even if it is approximately correct.
- (e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**
- (f) Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

Example: Finding angles and lengths using trigonometry

Markscheme			Candidates' Scripts	Marking
(a)	$\frac{\sin A}{3} = \frac{\sin 30}{4} \qquad (MI)(AI)$	$(a) \qquad \frac{\sin A}{4} = \frac{\sin 30}{3}$		(M1)(A0) (use of sine rule but with wrong values)
	$A = 22.0^{\circ} (22.0243)$ (A1)		and cannot be a	(A0) 1) here was not marked (ft) warded because there was in the same question part.)
(b)	$x = 7 \tan (22.0243^{\circ})$ (M1) = 2.83 (2.83163) (A1)(ft)	(b)	case (i) $x = 7 \tan 41.8^{\circ}$ = 6.26 case (ii) 6.26	(M1) (A1)(ft) (C0) since no working shown

4 **Using the Markscheme**

A marks are **dependent** on the preceding M mark being awarded, it is **not** possible to award (M0)(A1). Once an (M0) has been awarded, all subsequent A marks are lost in that part of the question, even if calculations are performed correctly, until the next M mark.

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The only exception will be for an answer where the accuracy is specified in the question - see section 5.

- A marks are dependent on the R mark being awarded, it is **not** possible to award (A1)(R0). Hence the (AI) is not awarded for a correct answer if no reason or the wrong reason is given.
- **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.

Where alternative methods for complete questions are included in the markscheme, they are indicated by 'OR' etc.

Unless the question specifies otherwise, accept equivalent forms. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$. (d)

On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.

Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:

the 3 significant figure answer worked through from full calculator display;

the exact value (for example $\sqrt{3}$ if applicable);

the full calculator display in the form 2.83163... as in the example above.

Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a different 3 significant figure answer, these solutions will also be given.

As this is an international examination, all valid alternative forms of notation should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; 1 · 7; 1,7.

Different descriptions of an interval: 3 < x < 5; (3, 5); (3

Different forms of notation for set properties (e.g. complement): A'; \overline{A} ; A^c ; U - A; $(A; U \setminus A)$.

Different forms of logic notation: $\neg p ; p'; \tilde{p} ; \overline{p} ; \sim p.$

 $p \Rightarrow q \; ; \; p \rightarrow q \; ; \; q \Leftarrow p \; .$

Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt an exception should be raised through scoris to the team leader.

As from Nov 11 the AP, FP and UP penalties will no longer apply. Accuracy and units will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

5 Accuracy of Answers

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the candidate's unrounded answer is seen and would round to the required 3 sf answer, then award (AI) and ignore subsequent rounding.

Note: The unrounded answer may appear in either the working box or on the final answer line.

- 2. If the candidate's unrounded answer is **not** seen then award (A1) if the answer given is **correctly** rounded to 2 or more significant figures, otherwise (A0).
- 3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarised in the table below and illustrated in the examples which follow.

	If candidates final answer is given						
	Exact or correct to 3 or more sf	Incorrect to 3sf	Correct to 2sf ³	Incorrect to 2sf	Correct or incorrect to 1sf		
Unrounded answer seen ¹	Award the final (A1) irrespective of correct or incorrect rounding						
Unrounded answer not seen ²	(A1)	(A0)	(A1)	(A0)	(A0)		
Treatment of subsequent parts	As per MS	Treat as follow through, only if working is seen. ³					

Examples:

Markschem	ie		Candidates' Scripts	Marking
9.43 (9.43398)	(A1)	(i)	9.43398 is seen in the working box followed by 9; 9.4; 9.43; 9.434 etc (correctly rounded)	(A1)
		(ii)	9.43398 is seen in the working box followed by 9.433; 9.44 etc (incorrectly rounded)	(AI)
		(iii)	9.4	(A1)
		(iv)	9	(A0) (correct to 1sf)
		(v)	9.3 (incorr	(A0) rectly rounded to 2sf)
		(vi)	9.44 (incorr	(A0) rectly rounded to 3sf)

Markschem	e		Candidates' Scripts	Marking
7.44 (7.43798)	(A1)	(i)	7.43798 is seen in the working box followed by 7; 7.4; 7.44; 7.438 etc (correctly rounded)	(A1)
		(ii)	7.43798 is seen in the working box followed by 7.437; 7.43 etc (incorrectly rounded)	(AI)
		(iii)	7.4	(A1)
		(iv)	7	(A0) (correct to 1sf)
		(v)	7.5 (incorr	(A0) ectly rounded to 2sf)
		(vi)	7.43 (incorr	(A0) ectly rounded to 3sf)

Example: ABC is a right angled triangle with angle $ABC = 90^{\circ}$, AC = 32 cm and AB = 30 cm. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
(a) BC = $\sqrt{32^2 - 30^2}$ (M1) Award (M1) for correct substitution in	(a) BC = $\sqrt{32^2 - 30^2}$	(M1)
Pythagorus' formula	11 (cm)	(A1)
=11.1 $(\sqrt{124},11.1355)$ (cm) (A1)	·	er only seen, but correct)
(b) Area = $\frac{1}{2} \times 30 \times 11.1355$ (MI)	(b) case (1) Area = $-\times 30 \times 11$	1 (M1)
Award (M1) for correct substitution in area of triangle formula		(working shown)
= $167(167.032)$ (cm ²) (A1)(ft)	$=165 (\mathrm{cm}^2)$	(A1)(ft)
10/(10/1002111)(0111)	case (ii) $= 165 (cm^2)$	(M0)(A0)(ft)
	, ,	shown, the answer 11 is no marks awarded here)

Rounding of an exact answer to 3 significant figures should be accepted if performed correctly. Exact answers such as $\frac{1}{4}$ can be written as decimals to less than three significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential.

Ratios of π and answers taking the form of square roots of integers or any rational power of an integer (e.g. $\sqrt{13}$, $2^{\frac{2}{3}}$, $\sqrt[4]{5}$,) may be accepted as exact answers. All other powers (e.g. of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a (ft) mark regardless of an immediately preceding (M0).

6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final A mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

Example: A financial question demands accuracy correct to 2dp.

Markscheme	Candidates' Scripts			Marking	
\$231.62 (231.6189)	(A1)	(i)	231.6		$(A\theta)$
		(ii)	232	(Correct roundin	(A0) g to incorrect level)
		(iii)	231.61		(A0)
		(iv)	232.00	(Parts ()	(A0) iii) and (iv) are both
					ling to correct level)

7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final *A* mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded (A1) under the accuracy rules given in Section 5.

Markscheme			Candidates' Scripts			Marking
(a)	37000m^2	(A1)	(a)	$36000\mathrm{m}^2$	(Incorrect answer so u	(A0) nits not considered)
(b)	3200 m ³	(AI)	(b)	$3200 \mathrm{m}^2$		(A0) (Incorrect units)

8 Graphic Display Calculators

Candidates will often obtain solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

BEFORE MARKING ANY SCRIPT YOU SHOULD CHECK WHETHER OR NOT THERE ARE ANY SUPPLEMENTARY SHEETS ADDED TO THE SCRIPT

(a)
$$z = \frac{12\cos(60^\circ)}{(4(8)+32)}$$

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Note: Award (M1) for correct substituted formula seen.

$$=0.09375\left(\frac{3}{32}\right) \tag{C2}$$

(ii)
$$0.0938$$
 (A1)(ft) (C1)

(iii)
$$9.375 \times 10^{-2} (9.38 \times 10^{-2})$$
 (A1)(ft)(A1)(ft) (C2)

Note: Award (AI)(ft) for 9.375, (AI)(ft) for $\times 10^{-2}$. Follow through from their part (a).

[6 marks]

QUESTION 2

$$(A1) \qquad (C1)$$

(b)
$$\frac{5+3+6+...+4}{13}$$

Note: Award *(M1)* for correctly substituted mean formula, division by 13 must be seen.

$$= 4.85 \left(\frac{63}{13}\right) \tag{4.84615...}$$

(d)
$$6-3.5$$
 (M1) $= 2.5$ (A1) (C2)

Note: Award (*M1*) for their quartiles seen or a correct ordered list. Accept a correct ordered list from any previous part of the question.

(a)	p	q	$\neg p$	$(p \wedge q)$	$(\neg p \veebar q)$	$(p \land q) \Rightarrow (\neg p \lor q)$
	Т	T	F	Т	T	Т
	T	F	F	F	F	T
	F	T	T	F	F	Т
	F	F	Т	F	Т	T

(A1)(A1)(A1)(ft)(A1)(ft)(C4)

Notes: Award (A1) for each correct column.

Award first (AI)(ft) from their third column in the table.

Award second (A1)(ft) from their fourth and fifth column in the table.

(b) (i) **Tautology** (A1)(ft)(C1)

Note: Answer must be consistent with last column in table.

(ii) All entries (in the final column) are true. (R1)(ft) (C1)

Note: Answer must be consistent with their answer to part (b)(i).

Note: Special case (A1)(R0) may be awarded.

[6 marks]

QUESTION 4

(a)
$$r = 0.814 (0.813745...)$$

(A2)(C2)

(b)
$$y = 0.888x + 13.5$$
 ($y = 0.887686...x + 13.4895...$)

(A1)(A1)

Note: Award (A1) for 0.888x, (A1) for 13.5. If the answer is not in the form of an equation award (A1)(A0).

OR

$$y - 63.2 = 0.888 (x - 56)$$

(A1)(A1)

(C2)

Note: Award (A1) for 0.888, (A1) for the correct means, \bar{x} and \bar{y} used.

(c)
$$y = 0.887686...(72) + 13.4895...$$

(M1)

Note: Award (M1) for 72 substituted into their equation of the regression line.

$$=77 (77.4028...)$$

(A1)(ft)

(C2)

Note: Accept a correct (ft) integer value or a decimal value which would round to the required 3 sf answer (ft). Follow through from their equation in part (b).

(a)
$$21+15+3-33$$
 or equivalent (M1)

Note: Award (M1) for correct use of all four numbers.

$$=6 (A1) (C2)$$

(b) (i)
$$q = 6$$
 (A1)(ft)

(ii)
$$p = 15, r = 9$$
 (A1)(ft) (C2)

Note: Follow through from their answer to part (a).

(c)
$$15+6+3$$
 (M1)

Note: Award (MI) for their figures seen in a correct calculation: 15+6+3 or 21+3 or 33-9

$$= 24 (A1)(ft) (C2)$$

Note: Follow through from parts (a) and (b) or from values shown on Venn diagram.

(a)
$$d = 321-326$$
 (or equivalent)
= -5 (A1)(A1) (C2)

Note: Award (A1) for negative sign. (A1) for 5.

(b)
$$S_{10} = \frac{10}{2} (2(326) + 9(-5))$$
 (M1)

Notes: Award *(M1)* for correctly substituted formula. Follow through from part (a).

OR

$$u_{10} = 281$$

$$S_{10} = \frac{10}{2}(326 + 281) \tag{M1}$$

Note: Award (M1) for correctly substituted formula, not for finding 281.

OR

If a list is seen award (M1) for the correct list of 10 terms consistent with their d.

(M1)

$$=3035 (A1)(ft) (C2)$$

Note: If d = 5 final answer is 3485. Follow through from part (a). No follow through if list used.

(c)
$$191 = 326 + (n-1)(-5)$$
 (or equivalent) (M1)

Notes: Award (*M1*) for correctly substituted formula. Follow through from part (a).

OR

If a list is seen award (MI) for the complete and correct list of terms or complete list of terms consistent with their d.

(M1)

$$n = 28 (A1)(ft) (C2)$$

Note: *n* must be a positive integer. Follow through from part (a). No follow through if list used.

(a)
$$\left(\frac{2+4}{2}, \frac{8+2}{2}\right)$$

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Note: Award (M1) for a correct substitution into the midpoint formula.

$$=(3,5) (C2)$$

Note: Brackets must be present for final (A1) to be awarded.

Note: Accept x = 3, y = 5.

(b)
$$\frac{8-4}{2-14}$$

Note: Award (M1) for correctly substituted formula.

$$= -\frac{1}{3} \left(\frac{-4}{12}, -0.333 \right) \quad (-0.3333333...) \tag{A1}$$

(c)
$$(y-5) = -\frac{1}{3}(x-3)$$
 (M1)(A1)(ft)

OR

$$5 = -\frac{1}{3}(3) + c$$
 (M1)
 $y = -\frac{1}{3}x + 6$ (A1)(ft) (C2)

Notes: Award (M1) for substitution of their gradient into equation of line with their values from (a) correctly substituted.

Accept correct equivalent forms of the equation of the line.

Follow through from their parts (a) and (b).

QUESTION 8 The first time the answer is not given to 2 decimal places the final (A1) in that part is not awarded, incorrect rounding, following correct method, can be ignored in subsequent parts.

(a) 1200×0.8154 (M1)

 $= 978.48 \, \text{EUR}$ (C2)

(b) 160×0.045 (M1)

 $= 7.20 \, \text{EUR} \tag{C2}$

(c) $152.80 \times \frac{1}{0.8202}$ (M1)

Note: Follow through from their answer to part (b).

=186.30 USD (A1)(ft) (C2)

Note: Follow through from part (b).

QUESTION 9 The first time a correct answer has incorrect or missing units, the final (A1) is not awarded.

(a)
$$V = \pi (15)^2 (12) + 0.5 \times \frac{4\pi (15)^3}{3}$$
 (M1)(M1)(M1)

Note: Award (M1) for correctly substituted cylinder formula, (M1) for correctly substituted sphere formula, (M1) for dividing the sphere formula by 2.

=
$$15550.8...$$

= $15600 \text{ m}^3 (4950\pi \text{ m}^3)$ (A1) (C4)

Notes: The final answer is 15600 m³; the units are required. The use of $\pi = 3.14$ which gives a final answer of 15500 (15543) is premature rounding; the final (A1) is not awarded.

(b)
$$SA = 0.5 \times 4\pi (15)^2$$
 (M1)
= 1413.71...
= 1410 m² (450 π m²) (A1) (C2)

Notes: The final answer is 1410 m²; do not penalize lack of units if this has been penalized in part (a).

(a) (i)
$$\frac{7}{11}$$
 (0.636, 63.6%) (0.636363...) (A1) (C1)

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(ii)
$$\frac{4}{10} \left(\frac{2}{5}, 0.4, 40 \% \right)$$
 (A1) (C1)

(iii)
$$\frac{28}{110} \left(\frac{14}{55}, 0.255, 25.5 \% \right) (0.254545...)$$
 (A1)(ft) (C1)

Note: Follow through from the product of their answers to parts (a) (i) and (ii).

(b)
$$\frac{28}{110} + \left(\frac{4}{11} \times \frac{7}{10}\right)$$
 OR $2 \times \frac{28}{110}$ (M1)(M1)

Notes: Award (M1) for using their $\frac{28}{110}$ as part of a combined probability expression.

(M1) for either adding $\frac{4}{11} \times \frac{7}{10}$ or for multiplying by 2.

$$= \frac{56}{110} \left(\frac{28}{55}, 0.509, 50.9 \% \right) (0.509090...)$$
 (A1)(ft) (C3)

Note: Follow through applies from their answer to part (a) (iii) and only when their answer is between 0 and 1.

(a)
$$I = \frac{200000 \times 15 \times 1.5}{100}$$
 (M1)

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Note: Award (M1) for correctly substituted simple interest formula.

$$=45000 \text{ INR}$$
 (C2)

(b)
$$A = 200000 \left(1 + \frac{15}{100 \times 12} \right)^{1.5 \times 12}$$
 (M1)(A1)

Note: Award *(M1)* for substituted compound interest formula, *(A1)* for correct substitutions.

$$= 250115.4788 \text{ INR}$$
 (A1)
= 250115 INR (A1) (C4)

Note: Award final (A1) for their answer correct to the nearest rupee.

(a)
$$u_9 = 512 \left(\frac{1}{4}\right)^8$$
 (M1)(A1)

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Notes: Award (M1) for substituted geometric sequence formula, (A1) for correct substitution.

OR

If a list is used, award (M1) for a list of 9 terms, (A1) for all 9 terms correct. (M1)(A1)

$$=\frac{1}{128} (0.0078125) \tag{C3}$$

Note: Award (A1) for exact answer only.

(b)
$$\frac{512\left(1 - \left(\frac{1}{4}\right)^n\right)}{1 - \left(\frac{1}{4}\right)} > 682.6$$
 (M1)(A1)(ft)

Notes: Award (MI) for setting substituted geometric sum formula > 682.6 (AI)(ft) for correct substitution into geometric sum formula. Follow through from their common ratio.

OR

If list is used, award (M1) for S(6) and S(7) seen, values don't have to be correct.

(A1) for correct S(6) and S(7).
$$(S(6) = 682.5 \text{ and } S(7) = 682.625)$$
. (M1)(A1)

$$n = 7 (A1)(ft) (C3)$$

Notes: Follow through from their common ratio.

Do not award the final (A1)(ft) if n is less than 5 or if n is not an integer.

(a) (i)
$$1.25 = -\frac{k}{2(2)}$$
 (M1)

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OR

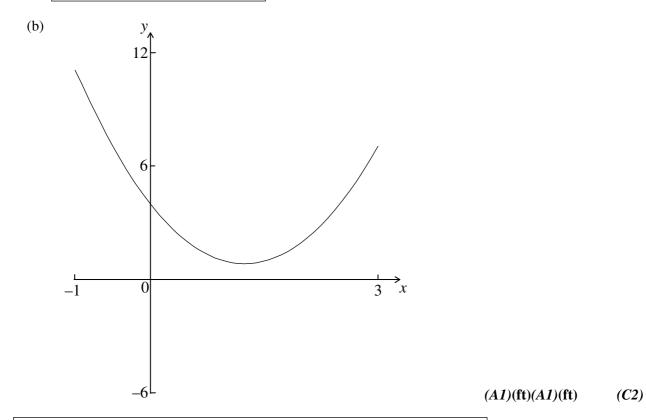
$$f'(x) = 4x + k = 0 (M1)$$

Note: Award (M1) for setting the gradient function to zero.

$$k = -5 (A1) (C2)$$

(ii)
$$2(1.25)^2 - 5(1.25) + 4$$
 (M1)
= 0.875 (A1)(ft) (C2)

Note: Follow through from their *k*.



Notes: Award (A1)(ft) for a curve with correct concavity consistent with their k passing through (0, 4).

(A1)(ft) for minimum in approximately the correct place.

Follow through from their part (a).

(a)
$$f'(x) = x^2 + 4x - 12$$
 (A1)(A1) (C3)

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Notes: Award (A1) for each term. Award at most (A1)(A1)(A0) if other terms are seen.

(b)
$$-6 \le x \le 2$$
 OR $-6 < x < 2$ (A1)(ft)(A1)(ft)(A1)

Notes: Award (A1)(ft) for -6, (A1)(ft) for 2, (A1) for consistent use of strict (<) or weak (\leq) inequalities.

Final (A1) for correct interval notation (accept alternative forms). This can only be awarded when the left hand side of the inequality is less than the right hand side of the inequality.

Follow through from their solutions to their f'(x) = 0 only if working seen.

[6 marks]

QUESTION 15

(c)

(a)
$$80 = 16 + k(c^0)$$
 (M1)
 $k = 64$ (C2)

(b)
$$48 = 16 + 64(c^{-2})$$

Note: Award (M1) for substitution of their k and (2, 48) into the equation for g(x).

$$c = \sqrt{2}$$
 (1.41) (1.41421...) (A1)(ft) (C2)

Notes: Award (M1)(A1)(ft) for $c = \pm \sqrt{2}$. Follow through from their answer to part (a).

$$y = 16 \tag{C2}$$

Note: Award (A1) for y = a constant, (A1) for 16.